Mechanics of the surf skimmer revisited

Takeshi Sugimoto
Kanagawa University, Yokohama 221-8686, Japan

(Received 24 September 2002; accepted 26 September 2002)

The mechanics of the surf skimmer, fun sport at the beach, is re-examined by using fundamental fluid mechanics. Comparison of the existing theories and consideration of the effects previously neglected lead to the conclusion: Edge's model is physically incorrect; Tuck and Dixon's theory provides physical insights into the surf skimming; there are several trade-offs in the mechanics of the surf skimmer and these make this sport fun and challenging.
[DOI: 10.1119/1.1523073]

INTRODUCTION

The surf skimmer is a toy used for sport at the beach. By jumping on a floating board, a player can skim along the water smoothly for several meters. In Ref. 1 Walker referred to Edge's work[2] to explain the mechanics of the surf skimmer. As early as the 1930s Green published a series of works analyzing a flat plate skimming the water surface by a discontinuous flow model.[3–5] His analytical results are expressed by elliptic functions and hence convoluted. Recently Tuck and Dixon[6] established a tractable method to solve Green's problem. The aim of this paper is to re-examine the mechanics of the surf skimmer using these new models and to show what constitutes a sounder basis on which to understand the physics behind this fun sport.

THEORY

Review of Edge's formulation

Figure 1 represents Edge's model, which describes the coordinate system and necessary nomenclature. A player with mass m skims the water on a board of length l tilted at an angle α to the horizontal. The acceleration due to gravity is denoted by g. The water of density ρ_w flows at velocity v and depth h infinitely upstream and downstream from the board; at the trailing edge the velocity and depth of the water become v_0 and h_0, respectively. The static pressure p takes the atmospheric pressure value p_0 at the trailing edge. The x axis is horizontal and positive upstream, with its origin at the trailing edge of the skimmer. The flow is assumed to be two dimensional, incompressible and inviscid; the effect of gravity on the water is neglected. The buoyancy forces upon the player and the board are neglected. Edge[2] states that the conservation of mass flux and Bernoulli's equation hold in the following forms, respectively,

$$hv = h_0 v_0 = \text{const} \tag{1}$$

and

$$p + \tfrac{1}{2}\rho_w v^2 = p_0 + \tfrac{1}{2}\rho_w v_0^2 = \text{const.} \tag{2}$$

By use of Eqs. (1) and (2) one can estimate the pressure difference on the board as

$$\begin{aligned} p - p_0 &= \frac{1}{2}\rho_w (v_0^2 - v^2) \\ &= \frac{1}{2}\rho_w v_0^2 \left(1 - \frac{h_0^2}{h^2}\right) \\ &= \frac{1}{2}\rho_w v_0^2 \left[1 - \frac{h_0^2}{(h_0 + \alpha x)^2}\right]. \end{aligned} \tag{3}$$

Integrating Eq. (3) along a board of width w, one obtains the vertical equilibrium condition

$$\begin{aligned} mg &= w \int_0^l (p - p_0)\mathrm{d}x \\ &= \frac{1}{2} w \rho_w v_0^2 \int_0^l \left[1 - \frac{h_0^2}{(h_0 + \alpha x)^2}\right]\mathrm{d}x \\ &= \frac{\rho_w v_0^2}{2} \frac{\alpha w l^2}{h_0 + \alpha l}. \end{aligned} \tag{4}$$

Edge approximates the rational term on the right-hand side of Eq. (3) by a truncated Taylor series so as to carry out the integration. Integration of the rational term, however, can be done exactly as shown above and the direct calculation leads us to the same final result as Edge.

On the other hand, the drag is given by the difference between the momentum of the water that is present on the upstream and downstream ends of the board in the control surface A in Fig. 1. This leads us to the equation for horizontal motion

$$\begin{aligned} m\frac{\mathrm{d}v}{\mathrm{d}t} &= w(\rho_w v^2 h - \rho_w v_0^2 h_0) \\ &= w\rho_w v_0^2 h_0 \left(\frac{h_0}{h_0 + \alpha l} - 1\right) \\ &= -\rho_w v_0^2 h_0 \frac{\alpha w l}{h_0 + \alpha l}. \end{aligned} \tag{5}$$

Using Eqs. (4) and (5) one reaches the final form of the equation for horizontal motion

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -2g\frac{h_0}{l}. \tag{6}$$

From this result Edge believes that the surf skimmer can go farther with a longer board in shallower water. According to Eqs. (5) and (6), the drag becomes zero if h_0 is zero. Is this true? That is one of the questions this paper tries to answer.

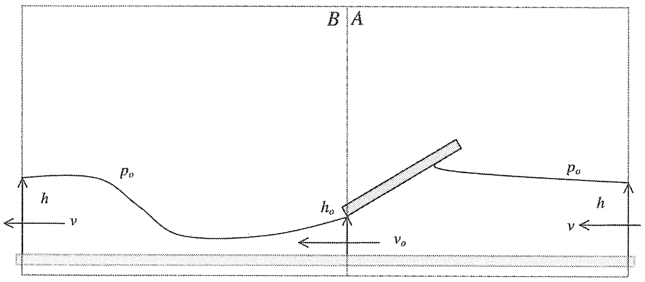

Fig. 1. Edge's model. Side view of the skimmer passing through the water. Note that there is recovery of water level in the far downstream of the skimmer.

A close examination of Fig. 1 reveals the rise of the water surface downstream of the board, but this particular flow never exists. Let us examine the momentum fluxes over the control surface B in Fig. 1. The gain of the momentum of the water incoming and outgoing through this control surface is equal to $\rho_w v_0^2 h_0 - \rho_w v^2 h$ and must balance with the pressure difference. But the pressure of the water is atmospheric everywhere on the control surface B, so the following must hold:

$$\rho_w v_0^2 h_0 - \rho_w v^2 h = 0. \tag{7}$$

Equation (7) states that the right-hand side of Eq. (5) is equal to zero. If h_0 is not zero, then the right-hand side of Eq. (4) is also equal to zero. The surf skimmer cannot lift up the rider. It is obvious there is no change in the momentum of the water, if we consider the control surfaces A and B altogether. That is, there can be no drag, since no momentum is transferred to the water.

A water flow in a channel sometimes exhibits a spontaneous increase in level known as a hydraulic jump. A tidal bore is an example of the hydraulic jump. One also observes a steady hydraulic jump at the downstream of a sluice gate but the level at the upstream of the sluice is much higher than the level of the hydraulic jump. The flow depicted in Fig. 1 has the same level at the infinite upstream and downstream of the board, but such a flow does not exist even within the realm of theory. To account for the hydraulic jump it is necessary to consider the effect of gravity.

Formulation for a flat plate skimming the water surface

Figure 2 shows the coordinate system and nomenclature for the Tuck and Dixon theory,[6] which is a special case of Green's model.[3–5] The flow is again assumed to be two dimensional, incompressible and inviscid, and effects due to gravity upon the flow are neglected. The board divides the flow into the spray, flying forward with thickness δ at infinity, and the wake, with depth h_∞ infinitely far downstream. The velocity of the spray returning upstream is found to be v at infinity. This and the continuity of mass flux leads us to

$$h = h_\infty + \delta. \tag{8}$$

The most important feature is the existence of the spray flying forward from the leading edge of the board. In Green's original model, the spray flies into the air at an angle. Although it is not entirely realistic, the theory implies that this spray flies infinitely far upstream into the air; the rest of this model is sound.

Tuck and Dixon introduce the contract ratio λ defined by

$$\lambda = \frac{h_\infty}{h_L}, \tag{9}$$

where h_L denotes the height of the stagnation point. They also assume the pressure and the horizontal velocity do not vary vertically, that is, the flow is one dimensional. This is justified, because the water is very shallow (1–2 cm). But the one dimensionality does not hold in the close vicinity of the stagnation point. The continuity of mass flux in the wake is

$$u_L h_L = v h_\infty$$

or

$$\frac{u_L}{v} = \frac{h_\infty}{h_L} = \lambda, \tag{10}$$

where u_L is the velocity through the vertical section at the stagnation point. These equations determine the relations between the variables at the stagnation point and the far downstream of the board.

We also need to know the relation between the variables at the stagnation point and the far upstream of the board. This can be done in the following manner. Applying Bernoulli's equation to the flow, except for the spray, we have

$$p_0 + \tfrac{1}{2}\rho_w v^2 = p_L + \tfrac{1}{2}\rho_w u_L^2$$

or

$$p_L - p_0 = \tfrac{1}{2}\rho_w (v^2 - u_L^2), \tag{11}$$

where p_L denotes the pressure in the vicinity of the stagnation point. The net force $(p_L - p_0)h_L$ has to balance the net momentum loss due to the spray thrown upstream, so one obtains

$$(p_L - p_0)h_L = \rho_w v^2 h - (-\rho_w v^2 \delta) - \rho_w u_L^2 h_L. \tag{12}$$

Using Eq. (11) the left-hand side of Eq. (12) becomes

$$\tfrac{1}{2}\rho_w (v^2 - u_L^2) h_L. \tag{13}$$

The right-hand side of Eq. (12) can be rewritten using Eqs. (8) and (10) to yield

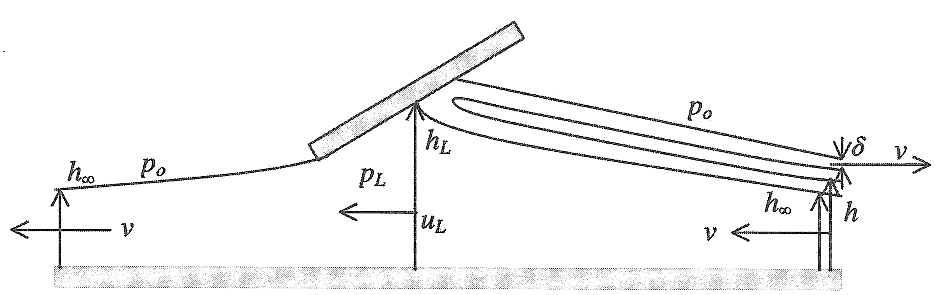

Fig. 2. Tuck and Dixon's model. Side view of the skimmer passing through the water. Note that a jet with thickness δ is returning toward the far upstream of the skimmer.

$$\rho_w v^2 h + \rho_w v^2 \delta - \rho_w u_L^2 h_L$$
$$= \rho_w v^2 h + \rho_w v^2 (h - h_\infty) - \rho_w u_L^2 h_L$$
$$= 2\rho_w v^2 h - \rho_w v u_L h_L - \rho_w u_L^2 h_L . \tag{14}$$

Equating Eq. (13) with Eq. (14), some algebra leads us to the final result

$$\tfrac{1}{2}\rho_w (v + u_L)^2 h_L = 2\rho_w v^2 h$$

or

$$\frac{h}{h_L} = \frac{(1+\lambda)^2}{4}. \tag{15}$$

This is the relation between the depth at the stagnation point and the depth far upstream of the board.

Another unique feature of the Tuck and Dixon analysis is the introduction of the wetted length l_w, which is the length between the stagnation point and the trailing edge of the board. These quantities define the angle of attack α

$$\tan\alpha = \frac{h_L - h_\infty}{l_w}. \tag{16}$$

Finally we shall derive the lift and drag acting on the surf skimmer. The explanation given below is different from Tuck and Dixon but the results are exactly the same.

The pressure under the board acts normal to the board. Therefore the total pressure force R also acts normal to the board. The component $R\sin\alpha$ is the pressure drag, while the component $R\cos\alpha$ is the lift. Since the drag balances the difference between the momentum of the water that is present on the upstream and downstream ends of the board, the following relation holds:

$$R\sin\alpha = \{\rho_w v^2 h - (-\rho_w v^2 \delta) - \rho_w v^2 h_\infty\} w$$
$$= 2\rho_w v^2 \delta w. \tag{17}$$

Using Eqs. (9), (15), (16), and (17), the lift can be written

$$R\cos\alpha = 2\rho_w v^2 \delta w \cot\alpha$$
$$= 2\rho_w v^2 (h - h_\infty) \frac{l_w}{h_L - h_\infty} w$$
$$= 2\rho_w v^2 \frac{h/h_L - h_\infty/h_L}{1 - h_\infty/h_L} l_w w$$
$$= \frac{1}{2}\rho_w v^2 A(1-\lambda), \tag{18}$$

where A is the wetted area and equal to $l_w w$.

Tuck and Dixon[6] derive Eq. (18) by integrating the pressure distribution, and hence they obtain the formula for the pitching moment as well. Using their formulas Tuck and Dixon successfully describe the hydrodynamics of a surf skimmer by the simplest physically correct model. In the next section we will discuss the effects that Tuck and Dixon omitted in their model.

Implications of the bow wave

The preceding two sections reveal the importance of displacing a portion of the incoming water towards somewhere other than downstream of the board. In reality there is no spray but instead a bow wave in the vicinity of the leading edge.

First we shall consider the effects of gravity upon the spray in Fig. 2. By taking account of gravity in Bernoulli's equation, one can estimate the possible height that the spray can fly up. Let Δh be this height from the water surface; one can apply Bernoulli's equation between the upper portion of the incoming water in the infinite upstream at depth d from the surface and the spray that stops flying at atmospheric pressure to obtain

$$p_0 + \tfrac{1}{2}\rho_w v^2 + \rho_w g d = p_0 + \rho_w g(\Delta h + d)$$

or

$$\Delta h = \frac{v^2}{2g}. \tag{19}$$

A typical initial velocity of the surf skimmer is several meters per second. If $v = 4.0$ m/s, then $\Delta h = 0.82$ m. If $v = 2.0$ m/s, then $\Delta h = 0.20$ m. These figures are too large for the height of the bow wave. Another difficulty in this scenario is the infinite thickness of the spray at $v=0$ because of the continuity of mass flux, i.e., $v\delta$.

If the flow is completely two dimensional, the incoming water steadily pushes the upper portion of the water upwards and forwards. In reality the board has a finite width and the upper portion of the water with thickness δ is also displaced sideways rather than purely forwards. This lateral flow and the forward flow may constitute the bow wave. To take account of three-dimensional effects properly we need to develop a lifting surface theory of wings with ground effect and free surface. Gravity and viscosity also play important roles in forming the bow wave. To understand their roles, thorough experiments including flow visualization are essential. These tasks are beyond the scope of this paper.

Other factors to be considered

Since two fluid media surround the player and the surf skimmer, both must be considered. The kinematic viscosity of air is 15 times larger than that of water. The aerodynamic Reynolds number for surf skimming is typically of order 10^5, while the hydrodynamic Reynolds number is of order 10^6.

The largest aerodynamic force is the drag acting on the player, because a standing man is a bluff body. The frictional drag on the board is much smaller than the human drag, because the board is a streamlined body. Practically, one may include the effect due to the frictional drag acting on the board in the human drag contribution. The aerodynamic lift forces acting on the player and the board are almost negligible compared to the hydrodynamic lift, and hence the induced drag due to lift is also negligible.

At the Reynolds number mentioned above the water is almost always turbulent in uncontrolled environments like a beach, and hence the friction drag due to the water is as large as the pressure drag.

Since the board is aerodynamically and hydrodynamically streamlined, the surface of the board should be smooth.

To close this section I would like to mention the possibility of lubrication by the water. Sometimes we observe that a sheet of paper slides for a long distance across a smooth floor. A very thin fluid layer is known to exert a normal stress much greater than a tangential stress. This phenomenon is accounted for by using the *lubrication model* of the Navier–

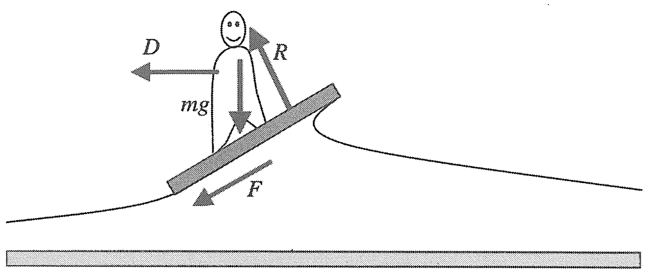

Fig. 3. Major forces acting on a player and a board. Note the directions of forces: the hydrodynamic pressure force (R); the weight (mg); the frictional drag (F); the aerodynamic drag (D).

Stokes equations (see details in Ref. 7, for example). To assure the validity of this model two conditions must be satisfied.

Let ϵ be h_∞ / l_w. Then one condition of lubrication model states

$$\epsilon \ll 1. \tag{20}$$

In case of the surf skimmer ϵ is at most of order 10^{-2}, and hence Eq. (20) holds.

Since the lubrication model is valid for laminar flow, another condition is

$$\mathrm{Re}\, \epsilon^2 \ll 1, \tag{21}$$

where

$$\mathrm{Re} = \frac{v l_w}{\nu_w}$$

and ν_w denotes the kinematic viscosity of water. If v is as slow as 1.0 m/s, then Re is of order 10^6. To satisfy Eq. (21) it is necessary that the trailing-edge height h_∞ should be less than 0.1 mm. Therefore we should not expect lubrication in the mechanics of the surf skimmer.

RESULTS AND DISCUSSION

Equations of motion

We shall derive the basic equations to describe the dynamics of the surf skimmer treated as a point mass. Figure 3 shows the major resultant forces acting on the player and the board. We assume there is no wind at the beach.

First we derive a vertical equilibrium condition. To simplify our analysis we shall estimate orders of related quantities and neglect less important ones. The largest known quantity is the player's weight, which we shall use as a measure of order estimation. The depth of the water h is typically several centimeters, so the vertical acceleration of the player and the board is much smaller than g and hence negligible. We neglect the buoyancy force on the board, or we assume that the weight of the board cancels out with the buoyancy force. The component due to the viscous friction is also negligible, because the angle of attack of the board is typically several degrees. Therefore the vertical equilibrium condition gives

$$mg = R \cos \alpha. \tag{22}$$

If we use Eq. (18) as the explicit expression for the lift, we have

$$mg = \tfrac{1}{2}\rho_w v^2 A (1-\lambda). \tag{23}$$

According to Eq. (23), it is necessary to make $A(1-\lambda)$ larger, as v becomes smaller: to stay afloat it is necessary to skim a great deal of water, as the velocity becomes smaller. There is, however, an upper bound for the thickness of the water, i.e., $\delta \leq h$. When the board hits the bottom of the water, λ becomes zero. Then the skimmer stops just after the velocity reaches the terminal value v_t given by

$$v_t = \sqrt{\frac{2mg}{\rho_w A}}. \tag{24}$$

This is equivalent to the lower bound of the velocity found by Tuck and Dixon.[6]

Next we derive the equation for horizontal motion, where we have three drag forces. The pressure drag $R \sin \alpha$ can be rewritten using Eq. (22) to be

$$R \sin \alpha = mg \tan \alpha. \tag{25}$$

The hydrodynamic friction F is given by

$$F = \tfrac{1}{2}\rho_w v^2 A c_f, \tag{26}$$

where c_f is the skin friction coefficient.

The aerodynamic drag D is given by

$$D = \tfrac{1}{2}\rho_a v^2 S_H C_D, \tag{27}$$

where ρ_a, S_H, and C_D denote the air density, the frontal area of the player, and the drag coefficient, respectively. Using Eqs. (25), (26), (27), and Newton's second law of motion, we obtain

$$m\frac{dv}{dt} = -mg \tan \alpha - \frac{1}{2}\rho_w v^2 A c_f \cos \alpha - \frac{1}{2}\rho_a v^2 S_H C_D, \tag{28}$$

or divided by m

$$\frac{dv}{dt} = -g \tan \alpha - \frac{1}{2m}\rho_w v^2 A c_f \cos \alpha - \frac{1}{2m}\rho_a v^2 S_H C_D, \tag{29}$$

where t denotes time.

The first term on the right-hand side of Eq. (29) is the physically correct expression of the right-hand side of Edge's Eq. (6).

Since no thrust acts on the surf skimmer as shown in Eq. (29), to ride the skimmer for a long distance it is necessary to reduce the drag forces in the following ways.

(1) To reduce the pressure drag it is necessary to keep the angle of attack as small as possible, but to be afloat it is necessary to tilt the board.
(2) To reduce the skin friction it is necessary to use a board with a hydrodynamically smooth lower surface, because as the velocity decreases a large area will be wetted.
(3) To reduce the aerodynamic drag it is necessary for the player to take a low posture.

Numerical results

To examine some quantitative aspects of the present analysis we derive the formula for the traveling distance of the surf skimmer. To integrate Eq. (29) we need further assumptions and estimates of parameters. The angle of attack is assumed to be kept constant. The skin friction coefficient is treated as a constant, i.e., 0.005. The aerodynamic drag coefficient is assumed to be 1.3. This value is insensitive to Reynolds number, because the player is a bluff body. The wetted area should be found as part of the solution. To solve the nonlinear problem as a whole set, one has to solve equations numerically. The aim of the present analysis is to obtain physical insights into the surf skimmer. We shall solve the problem approximately but analytically: we replace the wetted area A by the board area S_B. By the assumptions above, we are led to lower bounds for the terminal velocity v_t and the traveling distance. In this case Eq. (24) is reduced to

$$v_t \approx \sqrt{\frac{2mg}{\rho_w S_B}}. \tag{30}$$

Equation (29) becomes

$$\frac{dv}{dt} \approx -g\alpha - \frac{1}{2m}\rho_w v^2 S_B c_f - \frac{1}{2m}\rho_a v^2 S_H C_D. \tag{31}$$

If we let s be the traveling distance of the surf skimmer, then the following holds:

$$\frac{dv}{dt} = v\frac{dv}{ds}. \tag{32}$$

By replacing the left-hand side of Eq. (31) with the right-hand side of Eq. (32), one can integrate the differential equation by separation of variables. The constant of integration can be determined by the initial conditions $v(0)=v_0$ and $s(0)=0$ at $t=0$. One then obtains the total traveling distance, s_t, given by

$$s_t = s|_{v=v_t}$$
$$= \frac{m}{\rho_w S_B c_f + \rho_a S_H C_D} \times \ln\left|\frac{mg\alpha + (\rho_w S_B c_f + \rho_a S_H C_D)v_0^2/2}{mg\alpha + (\rho_w S_B c_f + \rho_a S_H C_D)v_t^2/2}\right|. \tag{33}$$

Let us examine the validity of Eqs. (30) and (33) by comparing the theoretical results with the experiment conducted by Edge. His son played on a surf skimmer on the beach. Edge took measurements in the field as well as analyzed 8 mm film. The details of the experimental data are the following: $m=29$ kg, $l=0.71$ m, initial velocity $=2.7$ m/s, terminal velocity $=0.45$ m/s, traveling distance $=4.9$ m, $\alpha=1.9$ degrees, initial height of the trailing edge $=5.1$ cm, and terminal height of the trailing edge $=2.5$ cm.

There is no data about the frontal area S_H, so we assume a 1.4 m tall and 30 cm wide boy. Densities are taken to be $\rho_w = 1025$ kg/m^3 and $\rho_a = 1.225$ kg/m^3.

Equation (30) gives

$$v_t = 1.2 \text{ m/s},$$

and using this estimate for v_t, Eq. (33) yields

$$s_t = 5.7 \text{ m}.$$

In principle Eqs. (30) and (33) should afford satisfactory results, although they are possibly simplistic. The relation between the terminal velocity and the traveling distance in the experiment contradicts the theoretical results. There are lots of unknown factors, winds, waves and so on, to be considered as possible causes of this discrepancy.

The next example is to show the relation between the initial velocity v_0 and the total traveling distance s_t. The assumed values are the following: $m=70$ kg, $S_H = 1.8 \times 0.4$ m^2, $S_B = (0.5)^2\pi$ m^2, and $\alpha = 2$ degrees. This situation is that of a 1.8 m tall adult who skims the water with a circular board (1 m in diameter). We also calculate the case of a child: $m=29$ kg, $S_H = 1.4 \times 0.3$ m^2, with the same board.

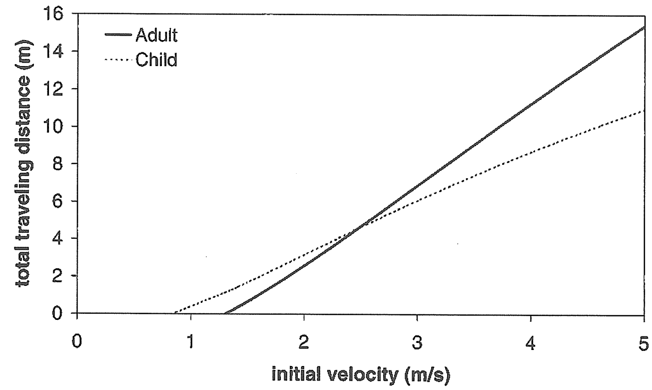

Fig. 4. Relations between initial velocity and total traveling distance. Comparison between the adult player and the child player with the same board.

By Eq. (30) one finds $v_t = 1.31$ m/s for the adult, and $v_t = 0.84$ m/s for the child. Substituting this v_t value into Eq. (33) we obtain the relation between v_0 and s_t as shown in Fig. 4. The heavier adult must start skimming at a faster initial velocity, while the lighter child can skim the water with a slower initial velocity.

Tuck and Dixon[6] state that the surf skimmer can travel for 5–10 m in 1–2 cm deep water and carry the player at speeds around 2–4 m/s. This statement is in fairly good agreement with the numerical results shown in Fig. 4. Our formulas should give the lower bound for v_t and s_t, because we replaced A by S_B and adopted larger values for c_f and C_D. It happened that overestimation balanced the neglected effects like the wave drag, the induced drag, and so on. This is an easy reason for the good agreement between the present simple formulas and the field observations, but another reason is closely related to the mechanics of the surf skimmer.

Figure 5 shows the relation between three drag forces and the velocity calculated using the same parameters as in Fig. 4. The largest drag is the pressure drag unless v is greater than around 3.5 m/s for the adult and 2.2 m/s for the child. The lift on the board must be equal to the player's weight. Therefore the pressure drag must be in proportion to the player's weight and does not depend on the velocity. During

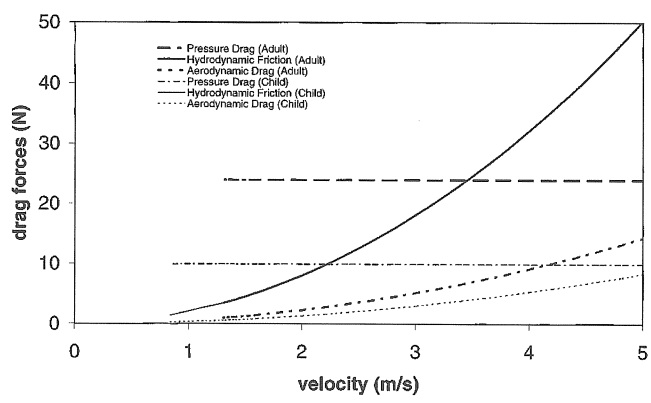

Fig. 5. Relation between various drag forces and the velocity. Comparison between the adult player and the child player with the same board.

most of the skimming time the pressure drag is the largest retarding force. This is another reason why the simplest formulas work well.

Figure 5 also explains the difference in the total traveling distance between the adult and the child. The pressure drag and the aerodynamic drag are dependent on the body weight and size, but the hydrodynamic friction is independent of the player. Therefore at higher speeds the friction becomes the major retarding force, and this situation works to the heavier adult's advantage.

Concluding remarks

We re-examined the mechanics of the surf skimmer by comparing the Edge and Tuck and Dixon models and considering several effects disregarded by these authors.

Edge's conclusion is found to be drawn from his physically incorrect model. It is not true that the surf skimmer can go farther with a longer board in shallower water.

One feature of surf skimming is concealed in the pressure drag. Since the lift has to be equal to the player's weight, the pressure drag becomes independent of the velocity. The pressure drag is a major retarding force in the usual operating range of the velocity.

There are several trade-offs in the mechanics of the surf skimmer.

(1) The angle of attack should be as small as possible to reduce the pressure drag, but the angle has to be nonzero to be afloat.
(2) The wetted area must become larger for the slower velocity, but the skin friction increases in proportion to the increasing wetted area.
(3) The player should take a posture as low as possible to reduce the aerodynamic drag, but the player has to keep balance and control the board.

These trade-offs make this sport fun and challenging.

By using Tuck and Dixon's model we derive the formulas that give the lower bounds for the terminal velocity v_t and the total traveling distance s_t, i.e., Eqs. (30) and (33). These formulas agree fairly well with the field observations.

Lubrication is found to be unimportant for the mechanics of the surf skimmer. There remain several questions to be answered in the future.

(1) The existence of the bow wave cannot be explained by the two-dimensional inviscid theory. To take account of lateral flow and wake it is necessary to develop a lifting surface theory. To account for the formation of the bow wave and the wake structure it is also necessary to consider the effects due to viscosity and gravity.
(2) To understand dynamics of surf skimming it is necessary to formulate the entire system as an interaction problem of rigid bodies, i.e., the player and the board, and surrounding flows.

In addition, well-controlled experiments are essential to understanding the physics behind the surf skimmer.

ACKNOWLEDGMENT

I thank Jim McElwaine, University of Cambridge, for his valuable suggestions and comments on my paper.

[1] J. Walker, *The Flying Circus of Physics* (Wiley, New York, 1977), pp. 89 and 257.
[2] R. D. Edge, "The surf skimmer," Am. J. Phys. **36**, 630–631 (1968).
[3] A. E. Green, "The gliding of a plate on a stream of finite depth," Proc. Cambridge Philos. Soc. **31**, 589–603 (1936).
[4] A. E. Green, "The gliding of a plate on a stream of finite depth. Part II," Proc. Cambridge Philos. Soc. **32**, 167–184 (1936).
[5] A. E. Green, "The moment of the fluid forces acting on a plate which is gliding on the surface of a stream," Proc. Cambridge Philos. Soc. **34**, 167–184 (1938).
[6] E. O. Tuck and A. Dixon, "Surf-skimmer planing hydrodynamics," J. Fluid Mech. **205**, 581–592 (1989).
[7] H. Ockendon and J. R. Ockendon, *Viscous Flow* (Cambridge U.P., Cambridge, 1995), pp. 64–70.