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ASPECTS OF SYMMETRIES IN MATHEMATICS

DISCOURSE ON THE *LATUS RECTUM* IN CONICS

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Abstract*: The geometric measures of conics are a 'latus rectum' and an 'eccentricity.' The latus rectum is a mathematical concept as well as an entity, e.g., a particular chord etc., embedded in a cone and its conic sections. The latus rectum has been consistently a length scale in conics, but its name and definition have been diversely changing from era to era for 2000 years: from 'ορθια' in Greek mathematics to a 'latus rectum' in Latin, and branching off with 'parameter'; the novel methods of constructing conics are shown by using the latera recta; natures of principal and non-principal latera recta are compared to understand modern definitions.*

Keywords: Synthetic Geometry, Latus Rectum, Parameter, Conics, History of Science.

1. INTRODUCTION

'Symmetry' literally means an integrated measure for a shape. This study focuses on a geometrical measure called a 'latus rectum' that literally means an 'erect side' in Latin. The conic sections, in short conics, are three kinds of quadratic curves playing important roles in natural science as well as engineering: the celestial bodies move in conic orbits; a burning mirror absorbs the solar heat to burn things at its focus; a whispering gallery is constructed in the form of ellipsoid; and so forth. The geometric measures in conics are a 'latus rectum' and an 'eccentricity': the latus rectum is a length scale, whilst the eccentricity is an aspect ratio. The latus rectum appeared almost at the same time that the treatises of conics were born in ancient Greek. On the other hand the eccentricity is a rather new idea. A winding history of the 'latus rectum,' in its meaning and definitions, is revealed by the present author's original scrutiny in classical texts: from east to west, and from ancient to modern times. It is important to understand a mathematical definition may vary from era to era, and hence it is dangerous for us to read old texts with our modern knowledge alone. We also show several new methods of constructing conics by using the latera recta to re-experience the ancient way of solving problems. In appendices we show the nature of non-principal latera recta.

2. HISTORICAL RESEARCH

A history of conics itself is concisely compiled in Coolidge's work (1945/1965). We shall tell the stories that did not appear in the former works with special emphasis upon the latus rectum. The latus rectum is a mathematical concept, *i.e.*, a length scale of a conic section, and at the same time the latus rectum is an entity, *i.e.*, a certain segment in a cone and its conic section. According to Coxeter (1989, p.116) "The chord (*LL'*) through the focus, parallel to the directrix, is called the latus rectum; its length is denoted by $2l$, so that $l = OL = a\ LH$." But this definition has only 200-year history, whilst the latus rectum itself was born 2000 years before. The latus rectum has many different faces.

2.1 Greco-Roman Antiquity

It is a common agreement that the discovery of conics is attributed to Menaechmus (c.380BC-c.320BC) (for example see Heath, 1931/2003): he wants to duplicate or halve the cube; if we have two lengths a and b and the mean proportionals x and y, we have $a : x = x : y = y : b$ that is to say $x^2 = ay$, $y^2 = bx$, and $xy = ab$; thus he encounters parabolas and a hyperbola; at this moment these curves have no names.

Aristaeus, the Elder (c.370BC-c.300BC) wrote five volumes of Solid Loci, but these are lost now. He identified three kinds of conics as shown in Fig.1: the section of the acute-angled cone; the section of the right-angled cone; the section of the obtuse-angled cone. Mathematicians in his era thought that cutting of a cone ought to be done at right angles to a generation of a cone.

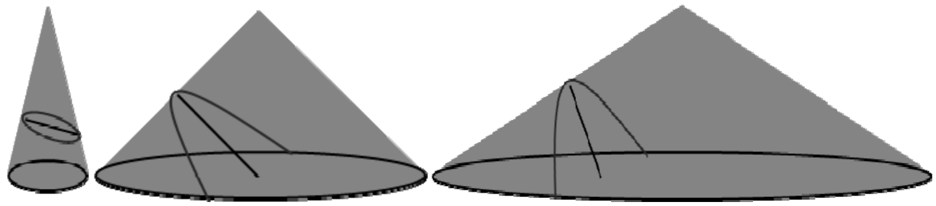

Figure 1. The left is the section of the acute-angled cone (ellipse); the middle is the section of the right-angled cone (parabola); the right is the section of the obtuse-angled cone (hyperbola).

Euclid (c.325BC-c.265BC) wrote four volumes of Conics, which were completed by Apollonius of Perga.

Archimedes (287BC-212BC) is the first person to find the principal latus rectum as an entity in the generating cone: what he says in Proposition III of On Conoids and Spheroids (Heiberg, 1880) is 'this scale is twice the distance from the principal vertex to the axis of the cone' as shown in Fig.2. On the other hand, he treats non-principal latera recta as mere length scales, *i.e.*, proportionals.

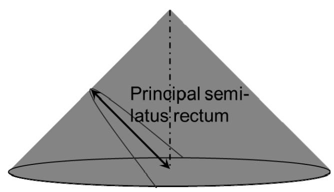

Figure 2. Archimedes' principal semi-latus rectum of a parabola in the generating cone.

Apollonius of Perga (c.262BC-c.190BC) wrote eight volumes of Conics (Memo, 1537; Maurolyco, 1548/1654; Commandino, 1566; Halley, 1710; Toomer, 1985; Roshdi et al., 2008-10). Volumes I-IV survive in Greek edited and commented by Eutocius of Ascalon (c.480-c.560), whilst Volumes I-VII survive in Arabic translation. Volume VIII is thought to be lost forever. Important is introduction of the nature called 'symptoma (Fried & Unguru, 1990)': the quadratic relations holding true in any oblique coordinate systems: it is explained as follows by use of the notations shown in Fig.3. The symptoma is the proportional relation that holds true between the conjugate diameters in such a manner that $Qv^2 : Pv\ vP = DC^2 : PC^2$, which is rewritten in an algebraic form:

$$Qv^2 = L\ Pv\ (1 \pm Pv/PG),$$

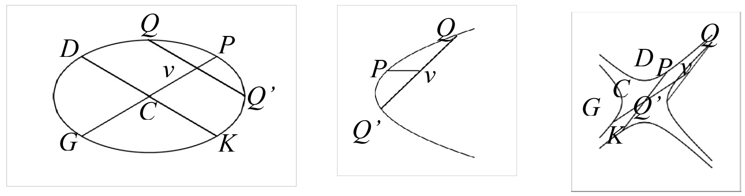

Figure 3. The conics and their symptomata: the quadratic relations in the oblique coordinate systems.

where L is the latus rectum defined by $2DC^2/PC$; the minus sign for an ellipse, the plus sign for the hyperbola, and PG = infinity for a parabola, respectively. When the conjugate diameters are the major and minor axes, L coincides with the latus rectum by the modern definition. As for the Ancient Greek Qv^2 is the square of a side Qv, whilst LPv is a rectangle with sides of L and Pv. Therefore Apollonius constructed figures like Fig.4. The latus rectum L has to be drawn perpendicular to the abscissa, and hence L is

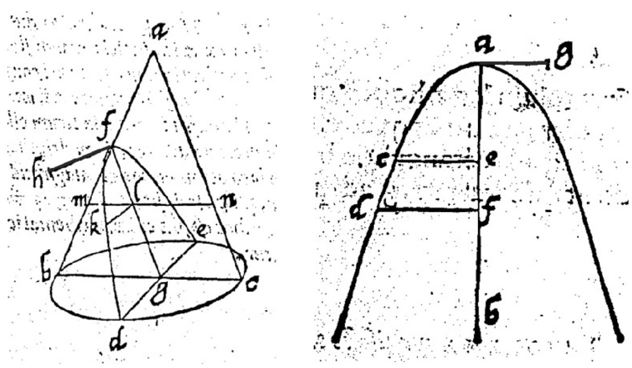

Figure 4. Latera recta in Apollonius' Conics (Commandino, 1566): the latus rectum *fh* is at right angles to the abscissa *fg* (left); the latus rectum *ag* is at right angles to the abscissa *ah* (right).

the 'erect side,' η ορθια πλευρα in Greek. At the dawn of Greek Mathematics the latus rectum is not a technical term but 'the straight line along with the abscissas constituting the rectangles which are compared to the squares of the ordinates.'

2.2 Islamic Golden-Age

Greek Mathematics was once demolished, but Islamic scientists salvaged the ancient wisdoms with their own language (Roshdi et al., 2008-10). There were Volumes I-VII of Apollonius' Conics but it was at first hard to understand completely for translation.

Fortunately Eutocius' annotated Volumes I-IV were discovered, and so by use of them Volumes I-IV and Volumes V-VII are translated into Arabic respectively by Hilal al-Himsi (?-883/884) and Thabit ibn Qurra (826-901) under the supervision of Ahmad (fl. 9C), one of the three Banu Musa. They used, as the latus rectum, the word 'al-dil al-qa'im,' which literally means 'upright side.' The Arabic Volumes I-IV are known to differ from those of Eutocius's Greek edition in several ways, *e.g.*, the order of propositions, the logic of proofs and so forth. There is another Arabic translation of Conics by an Ishaq, but this work is lost.

Ibn al-Haytham (965-1040), a.k.a. Alhazen, wrote On Completion of the Conics (Hodgendijk, 1985) to intend to reconstruct the lost Volume VIII of Conics. He knew a parabola having such a property as $x^2 = 4py$, where p is the distance between the principal vertex and the focus; $4p$ is the principal latus rectum of a parabola.

Ibn Sina (c.980-1037), a.k.a. Avicenna, is a Persian polymath. His construction of a parabola is illustrated in Fig.5 (left): draw a circle with its diameter AB equal to the principal latus rectum; draw the ordinate and the abscissa as their origin at A; draw a series of larger circles $BCED$ with the left tip B fixed, and produce parallel lines from the intersections C, D, and E among these circles and the coordinate axes to meet in two orthogonal intersections F and G: then shall the curve FAG be a parabola. The outline of its proof is enlightened by use of an oblique cone whose orthogonal vertical-sections are the isosceles right triangle and a parabola, FE^2 = (the principal latus rectum)AE: if $AB = AE = FE$, then the principal latus rectum is found to be equal to AB, that is the diameter of the circular section of the cone at the principal vertex of the parabola.

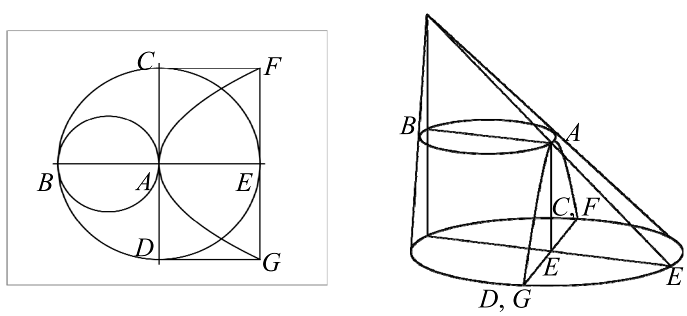

Figure 5. Ibn Sina's construction of a parabola: his construction (left) and its explanation (right) by use of an oblique cone; the same letters correspond in both the figures; the diameter AB is equal to the principal latus rectum.

2.3 Latin West

Strangely enough the treatise of conics returned to Europe via Arabic heritage (Clagett, 1980). Gerard of Cremona (c.1114-1187) translated the Arabic texts into Latin, and Witelo (fl. 13C) used them to construct his theory of perspective. William of Moerbeke (c.1215-c.1286) for the first time translated the Greek texts into Latin. In the context of optics several other people treated the theory of conics. The word 'latus rectum' was introduced in these years not as a technical term but the words to describe the situation of an 'erect side.' The word 'conic section' was introduced, of course in its Latin form, by Giorgio Valla (1447-1500) in his posthumous work (Valla, 1501).

The works mentioned above are not systematic but fragmental. The more authentic works were the translations of Apollonius' Conics in four volumes (Memo, 1537; Maurolyco, 1548/1654; Commandino, 1566). The word 'latus rectum' is used in their translations still as words to describe the situation of an 'erect side.'

In the Latin West the first original work on conics (Werner, 1522) is written by Johannes Werner (1468-1522). His construction of a parabola is entirely the same as ibn Sina's.

Claude Mydorge (1585-1647) is the first scholar to use the technical term for an 'erect side'; but in his work (Mydorge, 1631) he did not use a 'latus rectum' but coined the word 'parameter'; he also, for the first time, introduced the idea of 'principal': the word used in relation to the axis of symmetry; he defined 'assumed' for 'non-principal.' In his mind, different from the modern definition, all the three kinds of conics have the principal and assumed (non-principal) latera recta.

René Descartes (1596-1650) wrote his Geometry (Descartes, 1637), *i.e.*, the birth of analytic geometry; there appears the word 'coté droit principal (principal erect side).'

Another new line of study by use of conics is given birth to by French mathematicians Girard Desarques (1591-1661) and Blaise Pascal (1623-1662), that is to say projective geometry. To pursue this line is, however, beyond the scope of my present study.

Johannes Kepler (1571-1630) was well acquainted with conics in relation to optics but the most important is his discovery of three laws on the planetary motion: in particular the orbits of planets are ellipses. Isaac Newton (1642-1727) revealed the system of the world in his Principia (Newton, 1687, 1713, 1726); Newton proved that under the inverse square law of gravitation the orbits of the celestial bodies are three kinds of conics; Newton shows that the orbit of the great comet in 1680 is a parabola. Edmond Halley (1656-1742) was the editor and publisher of Newton's Principia, the first edition. He mastered the laws of gravitation, and he applied it to construct the treatise of the comets with the aid of historical materials. Thus Astronomy became a new field for conics. In 1710 Halley translated Apollonius' Conics into Latin from Greek (Vols.I-IV) and Arabic (Vols.V-VII), and he reconstructed the volume VIII based on Pappus' description in Collection (4C).

2.4 Far East

In ancient China geometry as science did not occur. The Chinese were interested in calendrical astronomy and measurements. In India the situation was similar.

Hulegu Khan, the leader of the Mongols, made the first encounter to Greek geometry. He conquered Bagdad and took Persian scientist Nasir al-Din al-Tusi (1201-1274). For Hulegu, al-Tusi constructed the observatory with the aid of Chinese astronomers; he wrote many commentaries on Greek mathematics including Archimedes and Apollonius, but the Chinese showed little interest in these works.

The authentic introduction of conics is made into China by the Jesuits: Ce liang quan yi (1631), 'The complete meaning of measurement' in English, was written in Chinese by Xu Guangqi (1562-1633) under the supervision of Matteo Ricci (1522-1610) and Jacque Rho (1593-1638); limited mention is made as that cutting a cone yields five kinds of figures: a triangle, a circle, an ellipse, a parabola, and a hyperbola. This Chinese book was bootlegged into Japan in 17C. Takakazu Seki (c.1641-1708), a Japanese mathematician, more independently studied conics, *e.g.*, calculating area of an ellipse and so forth, but there was no concept like the latus rectum.

The more systematic introduction of conics was made into Far East as recently as in the 19th century! At this moment the latus rectum was introduced via translation of the English texts into China and Japan: in 1866 Li Shanlan (1810-1882), with the aid of Joseph Edkins (1823-1905), translated Whewell (1846) into Chinese, and the word 焦弦 (the chord through the focus) was coined for the latus rectum in Chinese; in 1880 Tomochika Kawakita (1840-1919) translated Drew (1875) into Japanese, and the word 通径 (the diameter piercing the focus) was coined for the latus rectum in Japanese.

2.5 Modern Times

Newton's knowledge on conics differs from ours. The gap was born in 1802 when Jean-Baptiste Biot (1774-1862) wrote a book on quadratic curves (Biot, 1802). Since he is French, he uses the parameter for the latus rectum: in Art.59 (p.83) for the ellipse, $y = \pm B^2/A$ at $x = \pm\mathrm{Sqrt}(A^2 - B^2)$, or at the foci, where $2B^2/A$ is the parameter; in Art.72 (p.113) for the parabola, $y = p$ at $x = p/2$, or at the focus, where $2p$ is the parameter; in Art.84 (pp.137-138) for the hyperbola, $y = \pm B^2/A$ at $x = \pm\mathrm{Sqrt}(A^2 + B^2)$, or at the foci, where $2B^2/A$ is the parameter; Biot also mentioned that a parabola has a parameter for any oblique coordinate systems in Art.126 (pp.237-240). Today we know an ellipse and a hyperbola have only one latus rectum respectively, whilst a parabola has the principal latus rectum as well as non-principal latera recta. In an oblique coordinate system of a parabola the corresponding latus rectum is equal to the length of the chord running through the focus as shown in Appendix A. But in an oblique coordinate system of an ellipse and a hyperbola the corresponding latus rectum is not equal to the length of the chord running through the focus as disproved in Appendix B.

3. NOVEL METHODS OF CONSTRUCTING CONICS

Diorismos is the necessary and sufficient conditions for constructing a figure. We show novel methods of constructing conics based on given diorismos, the latus rectum inclusive. Nomenclatures used in the following figures are the same as those in Fig.3.

3.1 Synthetic Determination of the Latus Rectum

We shall show synthesis to find L the latus rectum for given Pv the abscissa, Qv the ordinate, and PG the diameter.

Lemma I: Parabola

Draw the square $ABCD$ in which each side is equal to Qv, and let E be on AB the point at which AE is equal to Pv; join ED, and from B produce a line parallel to ED to meet AD produced in F; then shall the length of AF be equal to L.

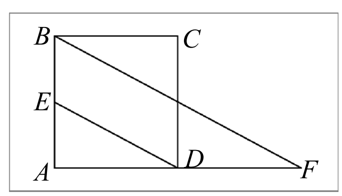

Figure 6. Finding L to satisfy $Qv^2 = L\ Pv$.

[Proof]

Since BF // ED, $\Delta EAD \cong \Delta BAF$ because of the AA Theorem; therefore $EA : AD = BA : AF$, or $Qv^2 = AF\ Pv$, and hence $AF = L$. [Q.E.D.]

Lemma II: Ellipse

We shall make most of Lemma I and Fig.6.

Produce BA, and let AG and AH be equal to vG and PG, respectively; join GF, and from H produce a line parallel to GF to meet AF produced in I; then shall the length of AI be equal to L.

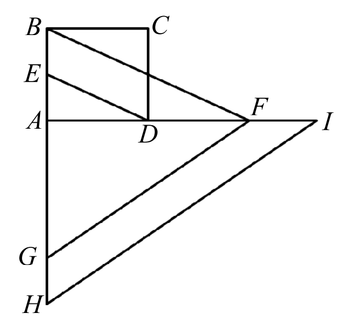

Figure 7. Finding L to satisfy $Qv^2 = L\ Pv\ vG/PG$.

[Proof]

Because of Lemma I, $Qv^2 = AF\ Pv$; since GF // HI, $\Delta AGF \cong \Delta AHI$ because of the AA Theorem; therefore $AF : AG = AI : AH$, or $AF = AI\ vG/PG$; eliminating AF from the relations above, we obtain $Qv^2 = AI\ Pv\ vG/PG$; therefore $AI = L$. [Q.E.D.]

Lemma III: Hyperbola

We shall make most of Lemma I and Fig.6, too.

Produce BA, and let AG and AH be equal to vG and PG, respectively; join GF, and from H produce a line parallel to GF to meet AF in I; then shall the length of AI be equal to L.

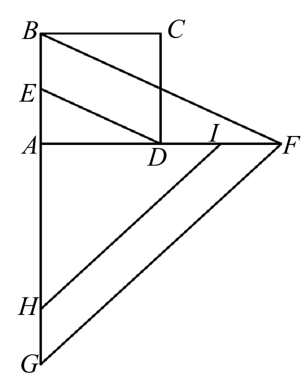

Figure 8. Finding L to satisfy $Qv^2 = L\ Pv\ vG/PG$.

[Proof]

Because of Lemma I, $Qv^2 = AF\ Pv$; since GF // HI, $\Delta AGF \cong \Delta AHI$ because of the AA Theorem; therefore $AF : AG = AI : AH$, or $AF = AI\ vG/PG$; eliminating AF from the relations above, we obtain $Qv^2 = AI\ Pv\ vG/PG$; therefore $AI = L$. [Q.E.D.]

3.2 Synthetic Determination of Conics in Arbitrary Coordinate Systems

Theorem I: Parabola

[Diorismos]

We are given the vertex *P,* the tangent at *P*, the diameter at *P*, and the latus rectum *L*.

[Construction]

Take a point *v* on the diameter, and let *Pv* be an abscissa; draw the chord through *v* parallel to the tangent at *P*; produce the diameter from *P* to *P'* such that *PP'* = *L*; take the point *O* in which a normal at *P* meets a line from *P'* at right angles to *PP'*; produce *OP* cutting the chord through *v* in *O'*; draw a circle *OO'* whose diameter is equal to the segment *OO'*; let *R* and *R'* be the points in which the circle *OO'* cuts the tangent at *P*; from *R* and *R'* draw lines parallel to *Pv* to meet the chord through *v* in *Q* and *Q'*, respectively; then shall the curve *QPQ'* be the required parabola.

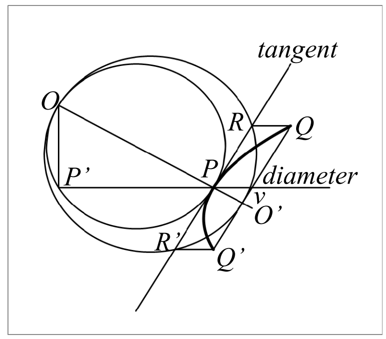

Figure 9. Construction of a parabola.

[Proof]

Δ*PvO'* ≅ Δ*POP'* because of the AA Theorem, and hence *Pv* : *PO'* = *PO* : *PP'*; applying the Power of a Point Theorem to the circle *ORO'R'*, we obtain *PR* : *PO'* = *PO* : *PR'*; since *RPR'* // *QvQ'*, *RP* : *PR'* = *Qv* : *vQ'*; eliminating *PO*, *PO'*, *PR*, and *PR'* from the relations above, we obtain *Pv* : *Qv* = *Qv* : *L*, that is $Qv^2 = L\ Pv$; therefore the curve *QPQ'* is the parabola with its latus rectum equal to *L*. [Q.E.D.]

Theorem II: Ellipse

[Diorismos]

We are given the vertex *P,* the tangent at *P*, the diameter *PG*, and the latus rectum *L*.

[Construction]

Take a point v on the diameter PG, and let Pv be an abscissa; draw the chord through v parallel to the tangent at P; from G draw a segment GN at right angles to PG and of length such that $GN = L$; join NP, and from v draw a line parallel to NP to cut GN in N'; produce the diameter PG to P' such that $PP' = GN'$; take the point O in which a normal at P meets a line from P' at right angles to PP'; produce OP cutting the chord through v in O'; draw a circle OO' whose diameter is equal to the segment OO'; let R and R' be the points in which the circle OO' cuts the tangent at P; from R and R' draw lines parallel to PG to meet the chord through v in Q and Q', respectively; then shall the curve QPQ' be the required ellipse.

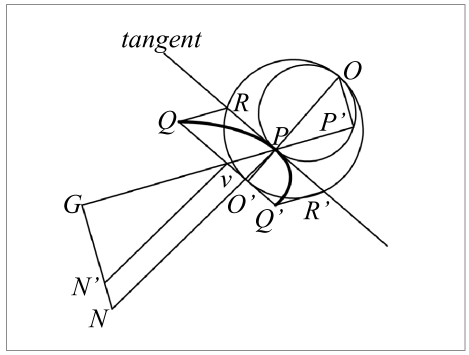

Figure 10. Construction of an ellipse.

[Proof]

Because of Lemma II $NG : N'G = PG : vG$, or $N'G = L\ vG/PG$; the rest of the argument is the same as the proof of Theorem I; hence we have the result in short: $Qv^2 = PP'\ Pv = N'G\ Pv = L\ Pv\ vG/PG$; therefore the curve QPQ' is the ellipse with its latus rectum equal to L. [Q.E.D.]

Theorem III: Hyperbola

[Diorismos]

We are given the vertex P, the tangent at P, the diameter PG, and the latus rectum L.

[Construction]

Take a point v on the diameter PG produced, and let Pv be an abscissa;

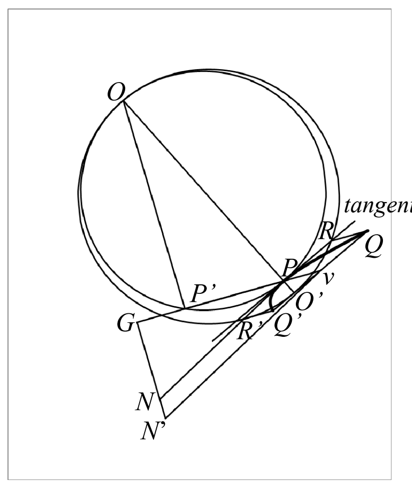

Figure 11. Construction of a hyperbola.

draw the chord through *v* parallel to the tangent at *P*; from *G* draw a segment *GN* at right angles to *PG* and of length such that *GN* = *L*; join *NP*, and from *v* draw a line parallel to *NP* to meet *GN* produced in *N'*; produce the diameter *PG* to *P'* such that *PP'* = *GN'*; take the point *O* in which a normal at *P* meets a line from *P'* at right angles to *PP'*; produce *OP* cutting the chord through *v* in *O'*; draw a circle *OO'* whose diameter is equal to the segment *OO'*; let *R* and *R'* be the points in which the circle *OO'* cuts the tangent at *P*; from *R* and *R'* draw lines parallel to *PG* to meet the chord through *v* in *Q* and *Q'*, respectively; then shall the curve *QPQ'* be the required hyperbola.

[Proof]

Because of Lemma III *NG* : *N'G* = *PG* : *vG*, or *N'G* = *L vG*/*PG*; the rest of the argument is the same as the proof of Theorem I; hence we have the result in short: $Qv^2 = PP'\ Pv = N'G\ Pv = L\ Pv\ vG/PG$; therefore the curve *QPQ'* is the hyperbola with its latus rectum equal to *L*. [Q.E.D.]

4. CONCLUSION

Here we summarise our findings:

(1) Transformation of meanings: from description of the situation (the erect side, or the upright side) to the technical term (the parameter, or the latus rectum); from proportionals in Symptomata to a length scale of a conic;

(2) Transformation of entities: from a segment or a diameter in a defining cone to a chord running through a focus in a conic;

(3) Discovered history: new stories about China and Japan are added;

(4) Novel construction: quite general methods are proposed;

(5) Non-principal latera recta: nature is revealed.

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APPENDIX A: LEMMA XIII IN BOOK I OF PRINCIPIA

The latus rectum of a parabola belonging to any vertex is four times the distance between the vertex and the focus.

It is evident from the theory of conics.

[Proof]

We are given the parabola, the vertex *P*, the tangent at *P*, and the focus *S*.

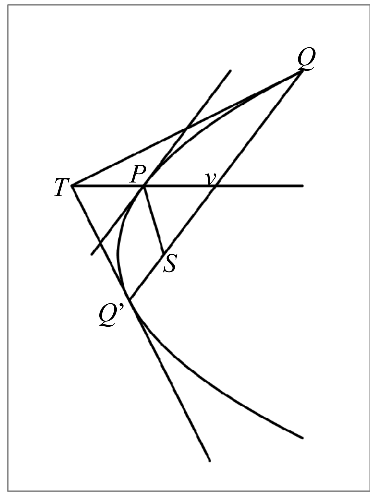

Figure 12. The non-principal latus rectum of a parabola.

Draw the tangents at *Q* and *Q'*; let *T* be the pole of these tangents above; *T* lies in the diameter *Pv* produced and of length such that *TP* = *Pv*, or *Tv* = 2*Pv* (it is prerequisite; see Sugimoto, 2009 for example); since ∠*vQT* = ∠*vTQ*, *Qv* = *Tv* = 2*Pv*; substituting 2*Pv* for *Qv* in the simptomata, we obtain $4Pv^2 = L\ Pv$, or *Pv* = *L*/4; accordingly *Qv* = 2*Pv* = *L*/2, and hence *QvSQ'* = 2*Qv* = *L*.

On the other hand *PS* = *Pv*, because ∠*PSv* = ∠*PvS*; therefore *L* = 4*PS*. [Q.E.D.]

APPENDIX B: NATURE OF NON-PRINCIPAL LATERA RECTA

I. Ellipse

The disproof is very simple. If *PG* = *DK*, then *L* = 2*PC* = 2*DC*; but *DK* the chord, whose length is equal to *L*, does not run through the focus.

The situation is shown in Fig.13 below. The short vertical segment is the principal latus rectum that run through the focus; $c = \text{Sqrt}(a^2 - b^2)$.

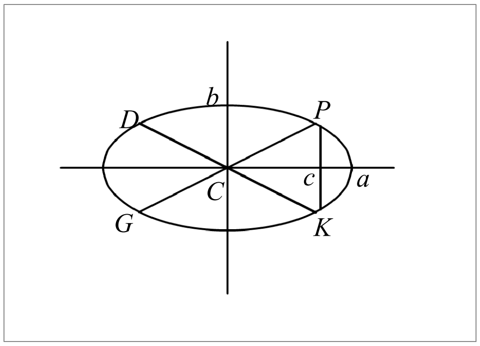

Figure 13. The non-principal latus rectum of an ellipse.

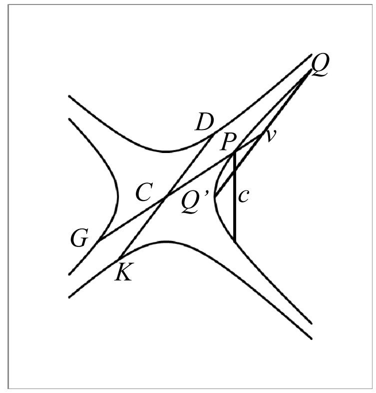

Figure 14. The non-principal latus rectum of a hyperbola.

II. Hyperbola

The disproof is the same as the case of an ellipse. If *PG* = *DK*, then *L* = 2*PC* = 2*DC*; but *QvQ'* the chord, whose length is equal to *L*, does not run through the focus.

The situation is shown in Fig.14 above in case of rectangular hyperbolas. The point *Q'* is at the principal vertex. The principal latus rectum is the short vertical segment adjacent the letter *c*, which indicates the location of the focus.

Thus in an ellipse or a hyperbola the chord, whose length is equal to a non-principal latus rectum, does not necessarily run through the focus. But in a parabola the chord, whose length is equal to a non-principal latus rectum, run through the focus as shown in Appendix A. That is why the modern definition eliminates the non-principal latera recta from an ellipse and a hyperbola.