A Spherical Horse Broadens Our Horizons of Study

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The spheres are the ultimate 3D toy models. Although the spheres are often treated as purely theoretical entities without real-world practicality, the spheres serve as hatchets to hard problems to be solved. Here are a couple of new twists to the classical mechanics: a sphere falling in the transverse winds and colliding with a branch. These results show usefulness of conceptual experiments by use of the spheres.
Key words: Spheres, Toy Models, Conceptual Experiment, Problem Solving

1. Introduction

There is an academic urban legend often referred to as "A Spherical Horse (Davis, 1998, for example)." The story is about a horse owner managing to make his horse win at the coming race. He hires three men, a chemist, a biologist, and a physicist, and orders them to improve the situation. Months later on the day before the race, the owner assembles the three men and asks the remedies. The chemist and the biologist explain their inventions. The physicist, however, begins his lecture by stating "Let us consider a spherical horse in simple harmonic motion ... "

However, in the course of solving a very hard problem, we often have recourse to changing the original problem into a general one or a special one, which would be solved more easily. This kind of tactics often leads us to the final solution of the original problem (Pólya, 2004). In such a detour toy models play important roles. The spheres are the ultimate 3D toy models to break through the curse of dimensionality. We shall show how spherical horses really work.

2. A Sphere Falling in the Transverse Winds

2.1 Motivation

Field observations reveal that the average flight range of the samaras is significantly larger than the conventional estimate: $D > Hw_T/U$, where D, H, w_T, and U designate the flight range, the height of releasing the samaras, the terminal velocity of the samaras, and the speed of the transverse wind, respectively; the excess is up to 10–20% of the conventional estimate. The most important point that the former works have neglected is the existence of the transverse winds. The winds release the samaras from their parent trees. We have carried out experiments in the transverse winds; we find that high lift acts on the samaras and that they fly farther than the conventional estimate (Ichikawa *et al.*, 2008). To annotate the results we want to consider a spherical samara.

2.2 Analytic results and discussion

We treat a falling body as a sphere of mass m and disregard any rotating motions. We assume moderate to high Reynolds number flow, and hence the aerodynamic drag is in proportion to the square of the total velocity relative to the transverse wind.

We use the coordinate system as shown in Fig. 1: x denotes the horizontal axis; z-axis is taken positive downward in the vertical plane. The wind is assumed to blow parallel to the positive x axis at the velocity U. To make our analysis concise, we assume the uniform wind. We observe from the frame fixed to the origin and define the velocity components u and w at the centre of gravity of the sphere.

Suppose a spherical samara starts to fall from the origin of the coordinate system. The drag has the horizontal and vertical components in proportion to the ratio of the respective velocity components to the total velocity. Thus we get the equations of motion with physical dimensions:

$$m\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{2}\rho S C_D (U-u)\sqrt{(U-u)^2+w^2},$$

and

$$m\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{1}{2}\rho S C_D w\sqrt{(U-u)^2+w^2},$$

where ρ, S, and C_D denote the air density, the reference area, and the drag coefficient, respectively. The initial conditions are given by $u(0) = w(0) = 0$.

We shall introduce the characteristic velocity and time to obtain the legible formalism. The terminal velocity w_T in free fall is the characteristic velocity given by $(2mg/\rho S C_D)^{1/2}$,while we define the characteristic time by w_T/g. Then we obtain the normalized and nondimensionalized equations of motion in the following vector form:

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{k} - |\mathbf{v}|\mathbf{v},$$

where

$$\mathbf{v} = (\lambda - u, w)^T,$$

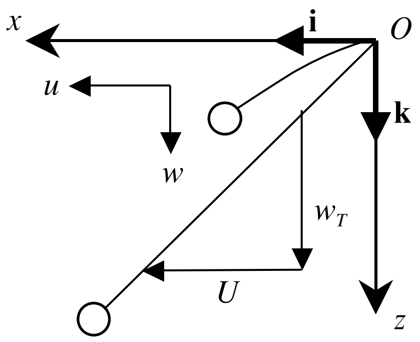

Fig. 1. Definition of the problem: A sphere falling in the transverse winds.

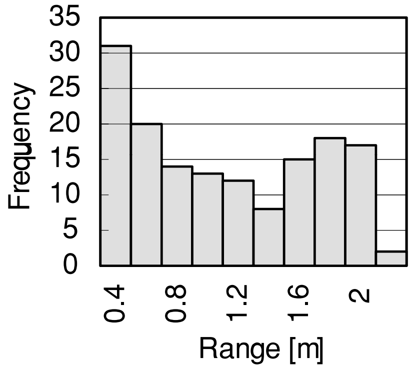

Fig. 2. Range distribution.

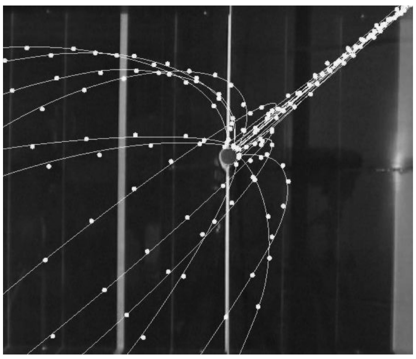

Fig. 3. Trajectories of spheres colliding with a circular bar.

and $\lambda = U/w_T$; in the equations above and hereafter u, w and t are used as nondimensional quantities; the initial condition is given by $\mathbf{v}(0) = \lambda\mathbf{i}$; the vectors $\mathbf{i}$ and $\mathbf{k}$ are the unit vectors in the horizontal and vertical directions, respectively. As shown above, the basic equation is a variation of Riccati equation in a two-dimensional vector form.

Initially $dz/dx = w/u = \lambda^{-2} < \lambda^{-1}$, if $\lambda > 1$. Since $dz/dx = \lambda^{-1}$ is the conventional formula, the initial locus comes above the straight line of the formula as shown in Fig. 1 for $\lambda > 1$.

Thus the *initial* mechanism is found to be the driving force of transporting samaras farther in case of the transverse wind of $U > w_T$.

3. A Sphere Colliding with a Circular Cylinder in the Transverse Winds

3.1 Motivation

Wind dispersal of samaras is affected by the existence of branches in forests. We tried examining this particular effect by the wind-tunnel experiments using spheres for samaras and a circular cylinder for a branch.

3.2 Experiments

The test section of our blow-down wind tunnel is 0.6 m high, 0.6 m wide, and 1.8 m long; we model the branch by a circular cylinder with 16 mm diameter placed transversely 0.34 m high from the floor and 1.5 m upstream from the end of the test section; we drop plastic spheres with 15 mm diameter through the hole on the wind tunnel ceiling where is 0.26 m high and 0.3 m upstream from our branch; wind velocity is fixed at $U = 5.0$ m/s.

The result is that our branch acts as a springboard rather than an obstacle: the flight range without the branch is on average 0.625 m with 0.037 m standard deviation, while the flight range with the branch is on average 1.11 m with 0.604 m standard deviation. Figure 2 shows the histogram of the flight range with the branch. It is apparent there are two peaks that stand for the flight ranges with upward and downward leaps. Figure 3 shows typical flight trajectories. Spheres leap markedly, although the coefficient of restitution is found to be as low as 0.347.

3.3 Annotation

We observed both top and back spins of spheres, and hence both positive and negative Magnus effects account for the double-peak distribution in the Range histogram, but this alone is not the entire cause of phenomena. The circular cylinder and its wake displace flow around it and hence trajectories of flying spheres, although quantitative explanation is not in our hands yet.

4. Conclusion

When we come across with problems complex in situations or geometries, it is quite useful to introduce spherical approximations to those problems. Such conceptual experiments surely lead us to divergent creative thinking, which then converges to the final solution.

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