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9 Introduction to some optimal techniques in bird soaring

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Abstract

The aim of this review is to re-examine some flight techniques, used by certain birds, which are thought to be optimal in an energy-saving sense. Static soaring may not seem to provide any new topics, but that is not true. There should be some explanations for phenomena like gull's wave riding near a ship, storm petrel's sea-anchor soaring and so on. One important aim of this review is to introduce, for the first time in English, Suzuki's thorough analysis of dynamic soaring on the basis of optimal control theory. Finally several challenges are summarized as problems to be answered in the future.

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1. Introduction

Berthold Laufer [1] pointed out in his monograph that the desire to fly is as old as mankind. He quotes from the Bible:

Oh that I have wings like a dove! For then would I fly away, and be at rest.

As this phrase applauds, our ancestors well realized that the bird is our master of flight. From the dawn of aviation to Wright brothers' success we have tried to understand how birds fly. Although several times we had strayed into the byways of magical levitation, balloons and what not, we came back to the royal road in the end. Leonardo da Vinci (1452-1519) studied the flight of birds and left sketches and notes, but unfortunately his manuscripts had been hidden until 1797. Around the turn of 19th century Sir George Cayley built hand-launched gliders. In Cayley's paper we find his comments and sketches related to aerodynamic forces acting on birds. He knew that, unlike birds, mankind could fly only with fixed wings. Otto Lilienthal, the father of gliders, studied the flight of birds thoroughly and published his findings as the definite book on bird flight [2]. He investigated how lift and drag work, why airfoil needs camber, and so on. The Wright brothers made most of Lilienthal's heritage to develop their powered, manned heavier-than-air aircraft. Their control system, the controversial patent, mimics how a bird twists its wings to change the flight course. One hundred years have passed since the Wright Flyer flew over Kitty Hawk; the last century may be called the age of applied aerodynamics, and its mainstream departed from studies of bird flight.

However a line remained uncut. From 1930s to early 1960s eminent ethologist Erich von Holst had studied animal flight by thorough and unique experiments. After his death in 1962 major part of his work was compiled in two volume books; its second volume covers most of his study on animal flight [3]. His collaboration with an aeronautical engineer is a classic in this field [4]. This paper still provides a lot of insights into zoological fluid dynamics.

After World War II many biologists, applied mathematicians, and engineers returned to the field of mechanics on animal locomotion. A partial list of related works, *e.g.*, Hertel [5], Gray [6], Pennycuick [7], Rüppel [8], Lighthill [9], Childress [10], and Vogel [11], may cover the early development of external biofluiddynamics, which is the term coined by Sir James Lighthill. Many conferences had also been held in this field in this era; see for example [12], [13]. In particular Lighthill was master of the subject. His commitment in this field resulted in many valuable papers that we can read now in the fourth volume of the collected works compiled by Hussaini [14]. Outcomes are understandings on scale effects, mechanics of wing flapping, novel phenomena like Weis-Fogh mechanism, and so forth.

In 1990s Alexander [15] and Tennekes [16] succeeded in attracting general readers' interest to animal flight, while Norberg [17] and Azuma [18] summarized broad knowledge on animal flight for specialists in an encyclopaedic manner. It is sure that biofluiddynamics gained the firm basis as a subject of science, and hence conferences have been held frequently, *e.g.*, [19].

Let us compare descriptions on flapping flight in the reviews by Lighthill [20] and Shyy *et al.* [21]. Lighthill surveys literatures up to 1990 from theoretical points of view, and hence he picks up topics related to Rayner's constant circulation model with vortex ring wake [22], [23]. On the other hand, Shyy *et al.* describe the non-steady, inviscid and flexible lifting surface model developed by Vest & Katz [24]. As examples of the state-of-the-art in numerical analysis of wing flapping I may add a couple of full Navier-Stokes simulations such as Smith [25] and Sun & Tang [26] that are applicable to flows with rather low Reynolds number up to the order of 10^3. The CFD lessons seem to me a new type of experimental tools. Theodorsen functions and Rayner's vortex-ring model, that is analytic models, may be surreal but reveal skeletal structures of physics behind the concerned phenomena. On the other hand, as summarized in Shyy *et al.* [21], the contemporary trend in this field is making for application to artefacts rather than understanding of Mother Nature, then CFD is indeed a powerful tool to assist engineers in designing wing-flapping machines.

Apart from application-oriented approach, other lines of investigation exist. For example we can list several literatures related to optimization of wing flapping: [27]-[30]; the authors of these papers look for lift distribution that maximizes aerodynamic thrust subject to the total lift constraint. The present review follows this line, and I shall consider bird flight by using the keyword, 'optimize.' Evolution itself is a series of stochastic and neutral phenomena, but survivors have fitted themselves to the environment to win the competition with rivals. Therefore I expect the paradigm shown in Fig.1 taken from [31]. Even in the field of evolutionary biotechnology engineering or rational design goes from top down to the targeted solution, while evolution is a bottom-up dynamical system leading to optimal function. I believe that the bird optimizes flight technique to lead advantageous life in severe environments.

What I try is to revisit some thought-to-be-solved problems and to re-examine physics behind the phenomena in yet another context. In some subsections I left several peculiar techniques as open questions. In Section 2 we show that control theory is a powerful tool to analyze special flight techniques used by birds. I treat the bird as a mass particle and discuss problems by use of equation of motion of the bird. Section 3 is used to summarize the revisits and make some comments on future challenges.

As for an extensive list of related literatures, I recommend readers to visit Rayner's Web site [32] where thorough lists are open to public and updated frequently.

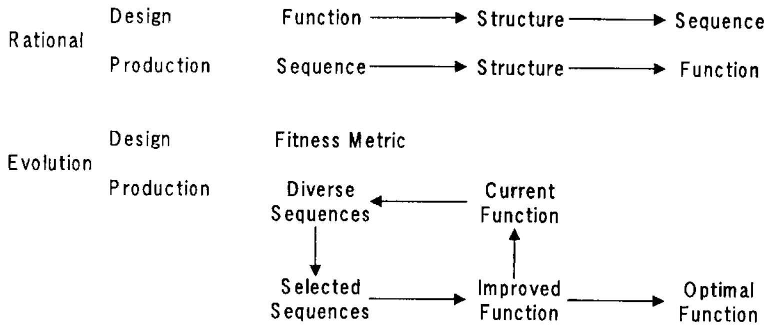

Figure 1. Design paradigm. redrawn from [31]

2 Soaring techniques

Throughout this section we use the conventional Cartesian coordinate: x, y and z axes correspond to the direction of flight, the horizontal and transverse axis to x, and the altitude, respectively.

2.1 Some special techniques in static soaring

There are several energy-saving techniques relatively less known and less trivial amongst static soaring.

Gray [6] wonders

why gulls soar on the leeward side of a steamer.

He conjectured three possibilities: use of thermal up-currents; use of obstructional currents; use of still air moving with the ship. I support the second possibility: gulls enjoy riding wavy air developed on the leeward side of the ship; they may be able to detect the invisible wave.

Figure 2 shows the schematic diagram for seagull's wave riding. We shall apply the mechanism of the surfing [33] to the wave-riding gull problem. Let us start our analysis from the following assumptions: (i) a wavy slope is moving with a ship at the constant speed V; (ii) the wavy slope does not change its surface shape designated by the smooth function $W_f(x)$; (iii) the coordinate system is fixed to the moving wavy slope; (iv) a gull of mass m is gliding along the wavy slope at the relative speed v in the tangent. The single and double primes designate the first and second derivatives with respect to x. The equations of motion in the tangent and the normal are respectively given by

$$m\dot{v} + \frac{1}{2}\rho v^2 SC_D + m\frac{gW_f'}{\sqrt{1+(W_f')^2}} = 0, \tag{1}$$

and

$$m\frac{v^2 W_f''}{\{1+(W_f')^2\}^{3/2}} - \frac{1}{2}\rho v^2 SC_L + m\frac{g}{\sqrt{1+(W_f')^2}} = 0, \tag{2}$$

where ρ, S, C_D, g, and C_L denote the air density, the wing area, drag coefficient, the gravity acceleration, and lift coefficient, respectively.

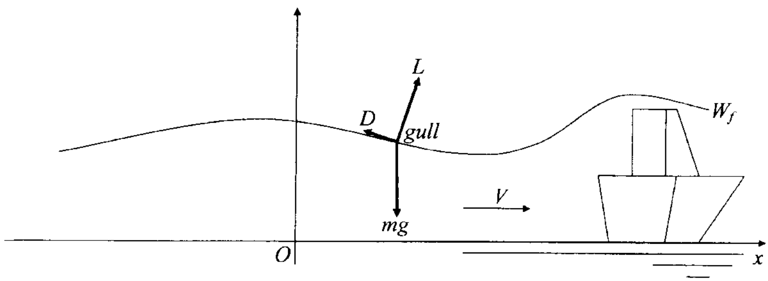

Figure 2. Schematic diagram for gull's wave riding near a moving ship. A ship is shown from its stern. A gull is soaring in the wave developed leeward the ship moving at the velocity V.

To stay with a wave the gull only needs to adjust the lift to fulfil Eq. (2). If $\dot{v} = 0$ in (1), we obtain

$$\begin{aligned} W_f' &= -\frac{\rho v^2 SC_D\sqrt{1+(W_f')^2}}{2mg} \\ &< 0, \end{aligned} \tag{3}$$

which constitutes a necessary condition for the existence of the steady state. Therefore wave riding is possible only on the front side of the wave. The equilibrium is stable if $\partial\dot{v}/\partial x < 0$. Differentiating (1) with respect to x, one obtains

$$\begin{aligned} \frac{\partial\dot{v}}{\partial x} &= -\{\frac{\partial v}{\partial x}\frac{\rho v SC_D}{m} + \frac{gW_f''}{\{1+(W_f')^2\}^{3/2}}\} \\ &< 0. \end{aligned}$$

If $\partial \dot{\upsilon}/\partial x < 0$, then $\partial \upsilon/\partial x$ is less than a constant value; suppose $\upsilon(x+dx) > \upsilon(x)$; if the bird is disturbed to move forward, it is transferred from the former steady state to another faster steady state there; this implies $\upsilon(x+dx) > \upsilon(x)$ to assure the stability of the steady state, and hence $\partial \upsilon/\partial x < 0$. Therefore the stability condition above yields

$$\begin{aligned} \frac{W_f''}{\{1+(W_f')^2\}^{3/2}} &> -\frac{\partial v}{\partial x}\frac{\rho v S C_D}{mg} \\ &> 0. \end{aligned} \tag{4}$$

That is, the equilibrium is stable only on the concave surface of the wave.

If the gull keeps the same position relative to the ship on the moving wave front, the following relation must hold:

$$v = V\sqrt{1+(W_f')^2}. \tag{5}$$

Using the first line of Eq. (3), one can eliminate $\rho S C_D/mg$ from the first line of Eq. (4); substituting Eq. (5) into that relation, some algebra leads us to

$$\begin{aligned} &(W_f')^2 < \frac{1}{2}, \\ &-\frac{1}{\sqrt{2}} < W_f' < 0. \end{aligned} \tag{6}$$

This range corresponds to possible glide angles, and the lower bound is around -35°.

Therefore gulls can soar stably on a lower part of the front side of a wave to go with a ship for a long distance.

Ships are also a source of things to eat for scavenging birds.

We shall move onto the next topic:

Storm petrels are known to walk on the sea surface by bringing aerodynamic and hydrodynamic forces under control.

This technique is called sea-anchor soaring (see for example [15]). Withers [34] analyzes the film and calculates the power requirement of sea-anchor soaring. Mechanically detailed analysis shows there are lower bounds about wind speeds for stable soaring [35]. Figure 3 shows the schema of sea-anchor soaring: the storm petrel of mass m stays aloft against the wind of speed U and walking on the water at speed V. The vertical forces must be in equilibrium:

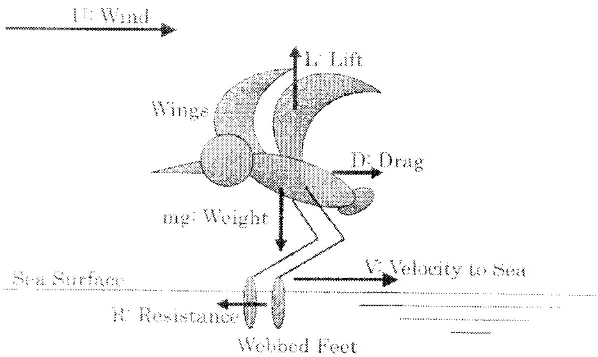

Figure 3. Schematic diagram for sea-anchor soaring, redrawn from [35]. A bird is soaring leeward in horizontal wind with its feet in the sea. All the forces acting on the bird are shown. Note the direction of U and V.

$$\frac{1}{2}\rho(U - V)^2 S C_L = mg. \tag{7}$$

To fly under this condition, the bird needs the wind faster than that required at the maximum lift coefficient $C_{L_{max}}$. . Let this lower bound of the wind speed be U_f, *i.e.* stall speed.

The horizontal forces are also in equilibrium:

$$\frac{1}{2}\rho(U - V)^2 S \left(C_{D_0} + \frac{C_L^2}{e\pi \mathit{AR}}\right) = \frac{1}{2}\rho_w A V^2 C_R, \tag{8}$$

where C_{D_0}, e, AR,ρw, A, and C_R denote zero-lift drag coefficient, the wing efficiency, the aspect ratio of the wing, the water density, the web area, and water resistance coefficient, respectively.

First we shall look for U_f in case of $C_L = C_{L_{max}}$; solving (7) with respect to V, we substitute it into (8) and then solve that with repect to U_f:

$$U_f = \sqrt{\frac{2mg}{\rho S C_{L_{max}}}} + \sqrt{\frac{2mg}{\rho_w A C_R C_{L_{max}}}\left(C_{D_0} + \frac{C_L^2}{e\pi \mathit{AR}}\right)}.$$

We can discuss the equilibrium in the following manner: eliminating C_L from (8) by use of (7), we have

$$a_0(U-V)^2 + a_1(U-V)^{-2} = bV^2, \tag{9}$$

where $a_0 = \rho SC_{D_0}/2$, $a_1 = 2m^2g^2/\rho Se\pi A$ and $b = \rho_w AC_R/2$.

Given U, we solve Eq. (9) with respect to V. Suppose we plot each side of Eq. (9) against V: the left-hand side diverges sharply at $V = U$, whilst the right-hand side is a parabola; and hence note that the equilibrium exists within the range of $V \in (0, U)$. In this range there are three possibilities: (9) has two, one or no roots. The existence of the equilibrium is assured if the wind speed is faster than that in case of one root. We define U_e as the wind speed that gives the single root to (9). When the curve on the left-hand side of (9) has only one point in common with the curve on the right-hand side of (9), the slope is also shared there. Therefore we take the first derivative of (9) with respect to V:

$$-2a_0(U-V) + 2a_1(U-V)^{-3} = 2bV. \tag{10}$$

Let us solve (9) and (10) with respect to V and U. After some algebra, we obtain

$$U_e = 4(b-a_0)\left[\frac{a_1}{2b(b-a_0)\left(b-4a_0+\sqrt{b(8a_0+b)}\right)\left(3b-\sqrt{b(8a_0+b)}\right)^2}\right]^{1/4}.$$

If $U_e < U$, there are two equilibria. The equilibrium at smaller V is stable, whilst the other is unstable. We shall discuss stability below.

According to Withers' data [34], Wilson's storm petrel *Oceanites oceanicus* has moderate wing area relative to their weight but large web area: $S = 0.017[\text{m}^2]$, $mg/S = 19.3[\text{N/m}^2]$, and $A = 0.0008[\text{m}^2]$. Using these data and the typical values for coefficients, we estimate the lower bounds of wind speed for Wilson's storm petrel: $U_e = 1.89$[m/s]; $U_f = 4.79$[m/s]. Since $U_f > U_e$, the storm petrel can always make most of sea-anchor soaring in the wind strong enoughto support its weight by the aerodynamic lift. Figure 4 summarizes the argument on the stability of the equilibrium: the upper inset shows water resistance and aerodynamic drag curves, labelled R and D, respectively; the schema below shows the balance between resistance and drag. Increasing V, the bird experiences less aerodynamic drag and more water resistance. Hence the bird is held back to the equilibrium. Decreasing V, the bird experiences more aerodynamic drag and less water resistance. Hence the bird is held back to the equilibrium. Therefore the equilibrium is stable. When the wind blows at $U = U_f$, the petrel's velocity to the water V is as slow as 0.256 [m/s]. It is easy for the petrel to walk back to its findings streaming away at such a slow speed.

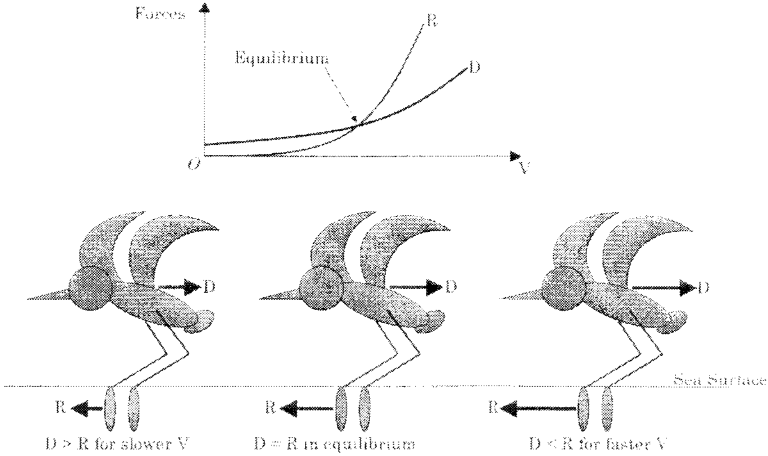

Figure 4. Stability of sea-anchor soaring, redrawn from [35]. Lower portion of this figure shows the equilibrium (in the middle of the upper inset) and two disturbed states. In the figure *D* and *R* designate aerodynamic drag and hydrodynamic resistance, respectively. A possible relation between resistance and drag is shown on top of this figure.

Some insects are known to become half airborne on water surface [36]. It is apparent that these insects adopt the same technique as sea-anchor soaring. To summarize

> In the wind blowing faster than the storm petrel's stall speed, hydrodynamic and aerodynamic forces always become in equilibrium to support the small webbed bird aloft at very small speed to the water.

In the end of this subsection I shall compile some peculiar techniques that are not yet annotated by theoreticians.

As studied by Withers & Timko [37], skimmers make most of the ground effect to feed with their scissors-like bills. They feed themselves on the wing with the under mandible skimming the water. Skimmers can flap their wings powerfully. During skimming they enjoy enhancement of lift and reduction of induced drag at the same time owing to the ground effect, so they flap their wings intermittently. Withers & Timko concluded the hydrodynamic drag acting on the mandible is negligible, but skimmer's feet sometimes dab the water. Therefore it is necessary to consider both hydrodynamics and aerodynamics of the skimming flight.

Diving birds are heavy and hence they rely on the ground effect to some extent, because that relaxes the power requirement as discussed in detail by

Rayner in [38]. Other interesting observation is diving petrel's underwater flight [39]: during foraging these birds glide in the water near the surface. To do so the lift must act downward. In this situation the ground effect in the water would repel the birds from the sea surface.

Prions are birds having specialized bills with comb-like lamellae in the upper mandible with which they filter-feed copepods. Their feeding technique is something special [40]:

> some use a peculiar technique called "hydroplaning", in which the bird rests lightly on the water and uses its feet to skim swiftly over the surface, with its wings outstretched, and the bill or even the whole head submerged.

2.2 Dynamic soaring

Mutton bird is one of Maori's delicacies. Its scientific name is *Puffinus griseus*, that is Sooty shearwater. In the 19th century Nature article the correspondent with the initials R.A. [41] calls *Diomedea melanophrys*, or Brown-browed mollymawk, as mutton bird. The size of this mollymawk is somewhere in between Wandering albatross Diomedea exulans and Sooty shearwater. This correspondent describes observations of sailing flight, which is now known as dynamic soaring. Later Rayleigh [42] correctly suggests that soaring would be possible in the non-uniform wind. In 1920s Idrac, supported by the French government, had studied dynamic soaring experimentally in the fields (see for example [43] and the comment [44]). This study provides detailed data that support Reyleigh's suggestion:

> albatrosses soar up windward and glide down leeward in the wind shear.

Idrac [45] reported two patterns of flight path in dynamic soaring: one is a figure of S, and another is a figure of α. Even Prandtl [46] describes his observation on the dynamic soaring of sea birds on the occasion of his ship trip from Yokohama to Honolulu and wondered how it is possible. Jameson [47] records the flight pattern of the albatross that had followed his battle cruiser for tens of hours without flapping. Cone [48] explains dynamic soaring by dividing into four phases and using two-dimensional argument. Vrana [49] is the first to use the variational principle to solve the three-dimensional flight of dynamic soaring but obtained the unrealistic and almost two-dimensional solution. This misleading result seems to arise from the assumption that the total energy relative to the wind, defined later in this subsection, must be constant. Wood [50] conducted the numerical analysis on the two-dimensional model by trial and error. Wilson [51] points out the possibility that albatrosses make most of wave draft developing along the ocean wave to take off. Sherwin [52] tried to apply dynamic soaring to manned aircraft but the discussion there is the use of the non-steady wind, *i.e.*, gust soaring defined later in this subsection.

It is Suzuki [53]-[56] that conducted the most thorough theoretical study on dynamic soaring. Here I will introduce his theory in English for the first time.

Albatross's position and velocity to the ground are respectively denoted by $\mathbf{r} = (x, y, z)$ and $\mathbf{u}$, while the wind shear is, without a loss of generality, given by $\mathbf{w} = (W, 0, 0)$.

Let us get started from the basics of soaring in the wind. Firstly we shall introduce the timederivative observed by a soaring bird along with the arc s of its flight-path:

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{D}t} &= \frac{\partial}{\partial t} + (\mathbf{u} \cdot \nabla) \\ &= \frac{\partial}{\partial t} + |\mathbf{u}|\frac{\partial}{\partial s}\end{aligned}$$

The instantaneous change in the kinetic energy of the bird is given by

$$(\mathbf{u} - \mathbf{w}) \cdot \frac{\mathrm{D}(\mathbf{u} - \mathbf{w})}{\mathrm{D}t}$$

If this is large enough to compensate for the dissipation due to aerodynamic drag and the loss of gravity-potential energy, it is possible for the bird to soar continuously without flapping its wings. If among the terms of the time-derivative of the kinetic energy above

$$-\mathbf{u} \cdot \frac{\mathrm{D}(\mathbf{w})}{\mathrm{D}t} > 0,$$

then the wind supplies energy to the bird. There are two possibilities. If

$$-u\frac{\partial W}{\partial t} > 0,$$

then the gust, the instantaneous change in the wind, becomes the source of the energy; we call the soaring technique using this effect *gust soaring.*

If

$$-u|\mathbf{u}|\frac{\partial W}{\partial s} > 0,$$

then the wind shear becomes the source of energy; we call the soaring technique using the effect due to the wind shear *dynamic soaring*. Over the ocean the wind shear has a nature, $\partial W/\partial z > 0$; therefore in a two-dimensional case, the criterion above affords a simple strategy:

Soar up windward. Glide down leeward.

This is just what Reyleigh [42] suggests and Idrac observed.

Let us move onto three dimensions. From now on, we concentrate to consider dynamic soaring alone, and hence we assume

$$\frac{\partial \mathbf{w}}{\partial t} = 0$$

to eliminate the effect due to gust soaring. We also assume without a loss of generality

$$W = W(z)$$

Figure 5 shows the coordinate systems. We shall introduce two frames to describe the motion of a bird flying in a sheared wind. The one is the frame **r** fixed to the ground. The other is the frame $\mathbf{R} = (X, Y, Z)$ fixed to the bird with its origin at the centre of gravity of the bird. The latter frame, which describes

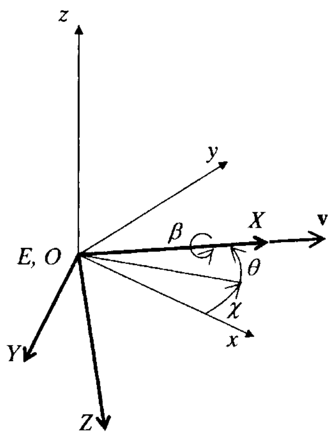

Figure 5. Coordinate systems for dynamic soaring. The origins are E for the frame **r** and O for the frame **R**. The ground frame **r** is successively rotated around the axes three times: rotation of **r** around x-axis 180 degrees to obtain the upside down frame $\mathbf{r}_0$; rotation of $\mathbf{r}_0$ around z_0-axis by the angle χ to obtain the frame $\mathbf{r}_1$; rotation of $\mathbf{r}_1$ around y_1-axis by the angle θ to obtain the frame $\mathbf{r}_2$; rotation of $\mathbf{r}_2$ around x_2-axis by the angle β to obtain the frame fixed to the bird **R**. Detailed description is summarized in Appendix A.

the attitude of the bird, is called an attitude coordinate set. The frame **R** is being drifted away from the origin of **r**. The transformation matrix **T** of vectors in the frame **r** to those in **R** is given by

$$\mathbf{T} = \begin{pmatrix} \cos\chi\cos\theta & -\sin\chi\cos\theta & \sin\theta \\ \cos\chi\sin\theta\sin\beta - \sin\chi\cos\beta & -\sin\chi\sin\theta\sin\beta - \cos\chi\cos\beta & -\cos\theta\sin\beta \\ \cos\chi\sin\theta\cos\beta + \sin\chi\sin\beta & -\sin\chi\sin\theta\cos\beta + \cos\chi\sin\beta & -\cos\theta\cos\beta \end{pmatrix}.$$

See Fig. 5 for the Euler angles χ, θ, β, and Appendix A for derivation of this matrix. Then, in terms of the bird velocity **v** relative to **R**, the bird velocity relative to the ground is written as

$$\mathbf{u} = \mathbf{T}^{-1} \cdot \mathbf{v} + \mathbf{w}. \tag{11}$$

In the frame fixed to the ground, the equation of motion of a bird of mass m is given by

$$m\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \mathbf{f}_I + m\mathbf{g}, \tag{12}$$

where $\mathbf{f}_I$ denotes the aerodynamic force and $\mathbf{g} = (0, 0, -g)$ is the gravity force.

Since the matrix **T** merely transforms components of vectors in frame **r** to those in frame **R**, the following is trivial

$$\mathbf{T} \cdot \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{\mathrm{D}}{\mathrm{D}t}\mathbf{T} \cdot \mathbf{u}.$$

Hence in frame **R**, Eq. (12) can be rewritten as

$$m\frac{\mathrm{D}}{\mathrm{D}t}\mathbf{T} \cdot \mathbf{u} = \mathbf{f} + m\mathbf{T} \cdot \mathbf{g}. \tag{13}$$

Here, $\mathbf{f} = \mathbf{T} \cdot \mathbf{f}_I = (-D, 0, -L)$, where the components D and L denote drag and lift, respectively.

The aerodynamic forces are written in the conventional form as

$$D = \frac{1}{2}\rho|\mathbf{v}|^2 S\left(C_{D_0} + \frac{C_L^2}{e\pi A\!\!R}\right),$$

$$L = \frac{1}{2}\rho|\mathbf{v}|^2 S C_L.$$

We shall define $\mathbf{s}(t)$, the path relative to the wind, by

$$\mathbf{s}(t) = \int_0^t \mathbf{T}^{-1} \cdot \mathbf{v}(t')\mathrm{d}t'.$$

Then $\mathbf{r}(t)$, the path observed in the ground frame, is given by

$$\mathbf{r}(t) = \mathbf{s}(t) + \int_0^t \mathbf{w}(t')\mathrm{d}t'.$$

Let us consider

$$E_G(t) \equiv \frac{1}{2}m|\mathbf{u}(t)|^2 - \frac{1}{2}m|\mathbf{u}(0)|^2 + mgz,$$

the margin of the total energy between the present and initial conditions observed in the ground frame, where z denotes the difference between the initial and present altitudes.

Taking the inner product of (12) with $\mathbf{u}$, we then integrate it with respect to t with the aid of (11). The result is

$$m\int_0^t \mathbf{u} \cdot \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t'}\mathrm{d}t' = \int_0^t (\mathbf{T}^{-1} \cdot \mathbf{f}) \cdot (\mathbf{T}^{-1} \cdot \mathbf{v})\mathrm{d}t' + \int_0^t (\mathbf{T}^{-1} \cdot \mathbf{f}) \cdot \mathbf{w}\mathrm{d}t' + m\int_0^t \mathbf{g} \cdot \mathbf{u}\mathrm{d}t'. \quad (14)$$

The left-hand side of (14) is equal to

$$\frac{1}{2}m|\mathbf{u}|^2 - \frac{1}{2}m|\mathbf{u}(0)|^2,$$

which is the margin of the kinetic energy between the present and initial conditions.

The following relation is trivial because of the nature of the orthogonal transformation matrix $\mathbf{T}$.

$$\int_0^t (\mathbf{T}^{-1} \cdot \mathbf{f}) \cdot (\mathbf{T}^{-1} \cdot \mathbf{v})\mathrm{d}t' = \int_0^t \mathbf{f} \cdot \mathbf{v}\mathrm{d}t'. \quad (15)$$

Therefore the first term on the right-hand side of (14) corresponds to the loss of energy due to the aerodynamic drag.

The second term on the right-hand side of (14) can be rewritten by use of (12) as

$$\begin{aligned}\int_0^t (\mathbf{T}^{-1}\cdot\mathbf{f})\cdot\mathbf{w}\mathrm{d}t' &= \int_0^t \left(m\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - m\mathbf{g}\right)\cdot\mathbf{w}\mathrm{d}t' \\ &= m\int_0^t \mathbf{w}\cdot\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t'}\mathrm{d}t', \end{aligned} \tag{16}$$

because $\mathbf{w} \perp \mathbf{g}$. The third term on the right-hand side of (14) is equal to $-mgz$. In all the right-hand side of (14) is rewritten in the following form:

$$\int_0^t \mathbf{f}\cdot\mathbf{v}\mathrm{d}t' + m\int_0^t \mathbf{w}\cdot\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t'}\mathrm{d}t' - mgz.$$

Therefore one reaches that

$$\begin{aligned} E_G(t) &= \frac{1}{2}m|\mathbf{u}(t)|^2 - \frac{1}{2}m|\mathbf{u}(0)|^2 + mgz \\ &= \int_0^t \mathbf{f}\cdot\mathbf{v}\mathrm{d}t' + m\int_0^t \mathbf{w}\cdot\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t'}\mathrm{d}t'. \end{aligned} \tag{17}$$

The first term on the right-hand side in the last expression (17) corresponds to the energy loss due to the aerodynamic drag. The second term denotes the energy brought by the wind, but its meaning is ambiguous. This term contains $\mathbf{u}$, which involves $\mathbf{w}$, so the cross term of $\mathbf{w}$ or the wind-ground interaction is involved. Therefore this term does not represent the pure energy gain from the wind.

Suzuki [53], [56] introduced a more sophisticated energy index,

$$E_A(t) \equiv \frac{1}{2}m|\mathbf{v}(t)|^2 - \frac{1}{2}m|\mathbf{v}(0)|^2 + mgz,$$

which he called the margin of the total energy relative to the wind.

Consider the work due to the bird motion relative to the wind, and one reaches that

$$\begin{aligned} m\int_0^t (\mathbf{u}-\mathbf{w})\cdot\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t'}\mathrm{d}t' &= m\int_0^t \left(\mathbf{T}^{-1}\cdot\mathbf{v}\right)\cdot\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t'}\mathrm{d}t' \\ &= m\int_0^t \left(\mathbf{T}^{-1}\cdot\mathbf{v}\right)\cdot\frac{\mathrm{D}}{\mathrm{D}t'}\left(\mathbf{T}^{-1}\cdot\mathbf{v}+\mathbf{w}\right)\mathrm{d}t' \\ &= \frac{1}{2}m|\mathbf{v}|^2 - \frac{1}{2}m|\mathbf{v}(0)|^2 + m\int_0^t \left(\mathbf{T}^{-1}\cdot\mathbf{v}\right)\cdot\frac{\mathrm{D}\mathbf{w}}{\mathrm{D}t'}\mathrm{d}t'. \end{aligned}$$

This leads us to

$$E_A(t) \doteq \frac{1}{2}m|\mathbf{v}(t)|^2 - \frac{1}{2}m|\mathbf{v}(0)|^2 + mgz$$
$$= \int_0^t \mathbf{f}\cdot\mathbf{v}\mathrm{d}t' - m\int_0^t \left(\mathbf{T}^{-1}\cdot\mathbf{v}\right)\cdot\frac{\mathrm{D}\mathbf{w}}{\mathrm{D}t'}\mathrm{d}t'. \quad (18)$$

The first term on the right-hand side in the last expression (18) is again the energy loss due to the aerodynamic drag, while the second term is an exact expression of the energy extracted from the wind.

The formalism for the optimal dynamic soaring is to maximize

$$E_A(t_f) = \int_0^{t_f} \mathbf{f}\cdot\mathbf{v}\mathrm{d}t - m\int_0^{t_f} \left(\mathbf{T}^{-1}\cdot\mathbf{v}\right)\cdot\frac{\mathrm{D}\mathbf{w}}{\mathrm{D}t}\mathrm{d}t, \quad (19)$$

where t_f denotes the free terminal time, subject to the equation of motion (12) and the inequality constraint, $0 \leq C_L(t) \leq C_{L_{max}}$ and $z_{\min} \leq z(t)$ to avoid dive into the sea.

The state variables are the heading angle $\chi(t)$, the pitch angle $\theta(t)$, the flight speed $|\mathbf{v}(t)|$, and the three components of the position vector, $x(t)$, $y(t)$, and $z(t)$, while the control variables are the roll angle $\beta(t)$ and the lift coefficient $C_L(t)$.

Initial conditions upon all the variables are given at $z(0) = z_{\min}$, while the terminal conditions are free except given $\chi(t_f)$, $\theta(t_f) = 0$, and $z(t_f) = z_{\min}$.

The important thing is the introduction of $E_A(t)$, not $E_G(t)$, because birds are in the wind not on the ground. Vrana assumed $E_A(t)$ to be constant. That is why his solution seems unrealistic.

Suzuki [53] solves the entire problem posed above upon many different initial conditions by the variational principle and numerical integration and finds many flight paths pretty similar to the field observations. He uses data of Wandering albatross: m = 9.0[kg], S = 0.5[m^2], AR = 18; the profile of the sheared wind is given by a power law as

$$W(z) = -6.9\left(\frac{z}{10}\right)^{1/6},$$

that is, the wind speed at the reference height z = 10[m] is −6.9[m/s]; $C_{D0} = 0.008$.

Figure 6 shows S-type flight pattern reproduced as a solution for $E_A(t_f)$, while Fig.7 shows α-type flight pattern of a solution; these constitute the

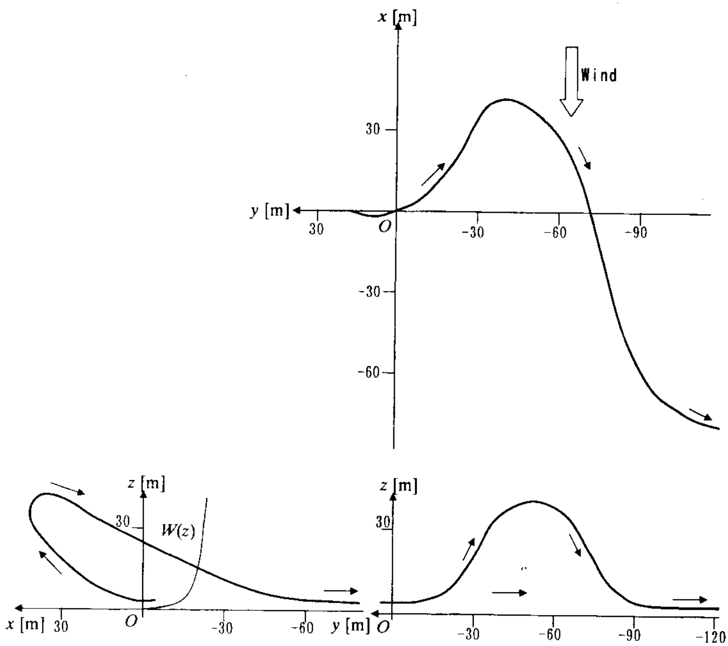

Figure 6. S-type flight path: the optimal solution to (19) for a Wandering albatross, redrawn from [53]: $m = 9.0$ [kg], $S = 0.5[\mathrm{m}^2]$, $C_{D_0} = 0.008$; important conditions given are $\chi(0) = \chi(t_f) = \pi/2$, $\theta(0) = \theta(t_f) = 0$, and $z(0) = z(t_f) = z_{\min}$.

optimal solutions with the identical initial and terminal conditions, but S-type is the global optimum; these are very similar to those observed by Idrac in the ocean. There are lots of different flight patterns in between these two; the optimum solutions transit from α-type to S-type flight patterns smoothly depending on the combination of $\chi(0)$ and $\chi(t_f)$, the direction to the wind.

One of the most interesting findings is that the global optimum of $E_A(t_f)$ is almost identical to the local optimum defined by the problem to maximize

$$\frac{\mathrm{d}E_A}{\mathrm{d}z} \equiv \frac{\mathrm{d}E_A}{\mathrm{d}t}|w(t)|^{-1}$$

subject to (12). The bird cannot know *a priori* how to maximize the total energy gain in one cycle of flight. What the bird can do at best is trying to

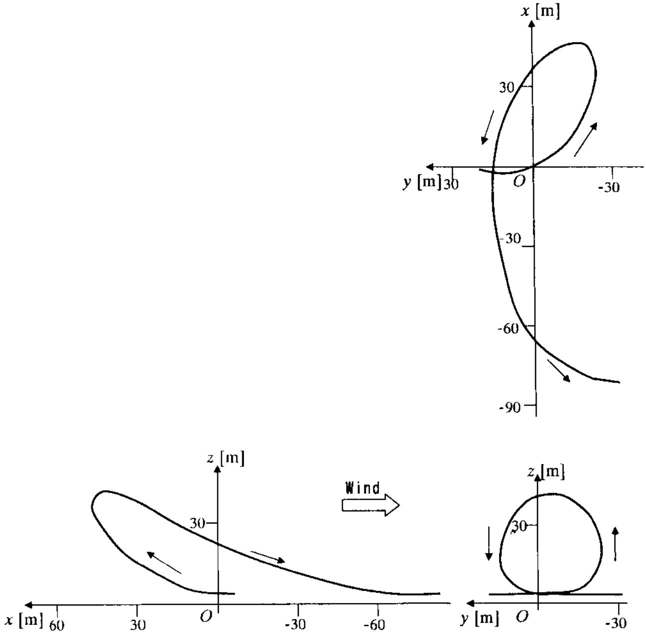

Figure 7. α-type flight path: the solution to (19) for a Wandering albatross, redrawn from [53]: $m = 9.0$[kg], $S = 0.5$[m^2], $C_{D_0} = 0.008$; important conditions given are $\chi(0) = \chi(t_f) = \pi/2$, $\theta(0) = \theta(t_f) = 0$, and $z(0) = z(t_f) = z_{min}$.

maximize the energy gain at every instance: that is the local optimization shown above.

Suzuki treats a bird as a particle, so it would be necessary to extend the theory using a rigid body model at least for more understanding and applying to artefacts. Nottebaum & Goebel [57] study the applicability of dynamic soaring to the flight technique for gliders, and the author would like to believe in their prospect: in the near future dynamic soaring will become available for manned aircraft.

3 Concluding remarks

The present review was a quick glimpse at some less known topics related to optimal techniques in bird flight. Bird flight keeps on providing the wide variety of riddles to be solved in the future.

One of the keys is feathers. Feathers play many roles: how primaries can generate so much lift and thrust, how coverts prevent separation, the role of wing tip slots is still an open question, asymmetry of vanes also needs thorough examination, etc. All these things are not yet well understood.

We should also pay attention to the linkage between mechanics and physiology. This is the field where external and internal biofluiddynamics meet together.

Yet another line of new study may go to ecology. Diffusion process is closely related to flock dynamics of birds [58], [59]. It would be interesting to look for a bridge from heuristics of *boids* [60], [61] to ecological diffusion problems; the former is a bottom-up approach, while the latter is a top-down approach. In the ocean, foraging flights of sea birds provide ecological diffusion problem and are strongly linked to meteorological pattern formation. Birds of passage are also the objects of this line of study.

Novel devices and techniques make field observation more quantitative and accurate. Hence experiments reveal the new facts that will become challenges for theoreticians to explain why and how. We will learn a lot of lessons from bird flight.

I would like to thank Susumu Kobayashi for his valuable comments on my first draft.

Appendix A. Transformation from the frame fixed to the ground to the frame fixed to the bird

The frame fixed to the ground is expressed by $\mathbf{r} = (x, y, z)$. The frame fixed to the bird is expressed by $\mathbf{R} = (X, Y, Z)$.

First we rotate $\mathbf{r}$ around x-axis 180 degrees to obtain the upside down frame $\mathbf{r}_0$:

$$\mathbf{r_0} = \mathbf{F_0} \cdot \mathbf{r},$$

where

$$\mathbf{F_0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Now we shall describe the three processes of talking the Euler angles. We rotate $\mathbf{r}_0$ around z_0-axis by the angle χ to obtain the frame $\mathbf{r}_1$:

$$\mathbf{r_1} = \mathbf{F_1} \cdot \mathbf{r_0},$$

where

$$\mathbf{F_1} = \begin{pmatrix} \cos\chi & \sin\chi & 0 \\ -\sin\chi & \cos\chi & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then we rotate $\mathbf{r}_1$ around y_1-axis by the angle θ to obtain the frame $\mathbf{r}_2$:

$$\mathbf{r_2} = \mathbf{F_2} \cdot \mathbf{r_1},$$

where

$$\mathbf{F_2} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}.$$

Lastly we rotate $\mathbf{r}_2$ around x_2-axis by the angle θ to obtain the frame $\mathbf{R}$:

$$\mathbf{R} = \mathbf{F_3} \cdot \mathbf{r_2},$$

where

$$\mathbf{F_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix}.$$

Therefore the matrix $\mathbf{T}$ transforming from $\mathbf{r}$ to $\mathbf{R}$ is given by

$$\mathbf{T} = \mathbf{F_3} \cdot \mathbf{F_2} \cdot \mathbf{F_1} \cdot \mathbf{F_0},$$

or

$$\mathbf{T} = \begin{pmatrix} \cos\chi\cos\theta & -\sin\chi\cos\theta & \sin\theta \\ \cos\chi\sin\theta\sin\beta - \sin\chi\cos\beta & -\sin\chi\sin\theta\sin\beta - \cos\chi\cos\beta & -\cos\theta\sin\beta \\ \cos\chi\sin\theta\cos\beta + \sin\chi\sin\beta & -\sin\chi\sin\theta\cos\beta + \cos\chi\sin\beta & -\cos\theta\cos\beta \end{pmatrix}.$$

We derive the relation between the angular velocity vector $\boldsymbol{\Omega}$ in the frame $\mathbf{R}$ and the angular velocities of the Euler angles $\dot{\chi}, \dot{\theta}$ and $\dot{\beta}$ in the following manner.

$$\boldsymbol{\Omega} = \mathbf{F_3} \cdot \mathbf{F_2} \cdot \mathbf{F_1} \cdot \boldsymbol{\omega_1} + \mathbf{F_3} \cdot \mathbf{F_2} \cdot \boldsymbol{\omega_2} + \mathbf{F_3} \cdot \boldsymbol{\omega_3},$$

where $\omega_1 = (0, 0, \dot{\chi}), \omega_2 = (0, \dot{\theta}, 0)$, and $\omega_3 = (\dot{\beta}, 0, 0)$. Hence we obtain

$$\boldsymbol{\Omega} = \mathbf{T}_\omega \cdot \boldsymbol{\omega},$$

where

$$\mathbf{T}_\omega = \begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\beta & \cos\theta\sin\beta \\ 0 & -\sin\beta & \cos\theta\cos\beta \end{pmatrix},$$

and $\omega = (\dot{\beta}, \dot{\theta}, \dot{\chi})$.

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